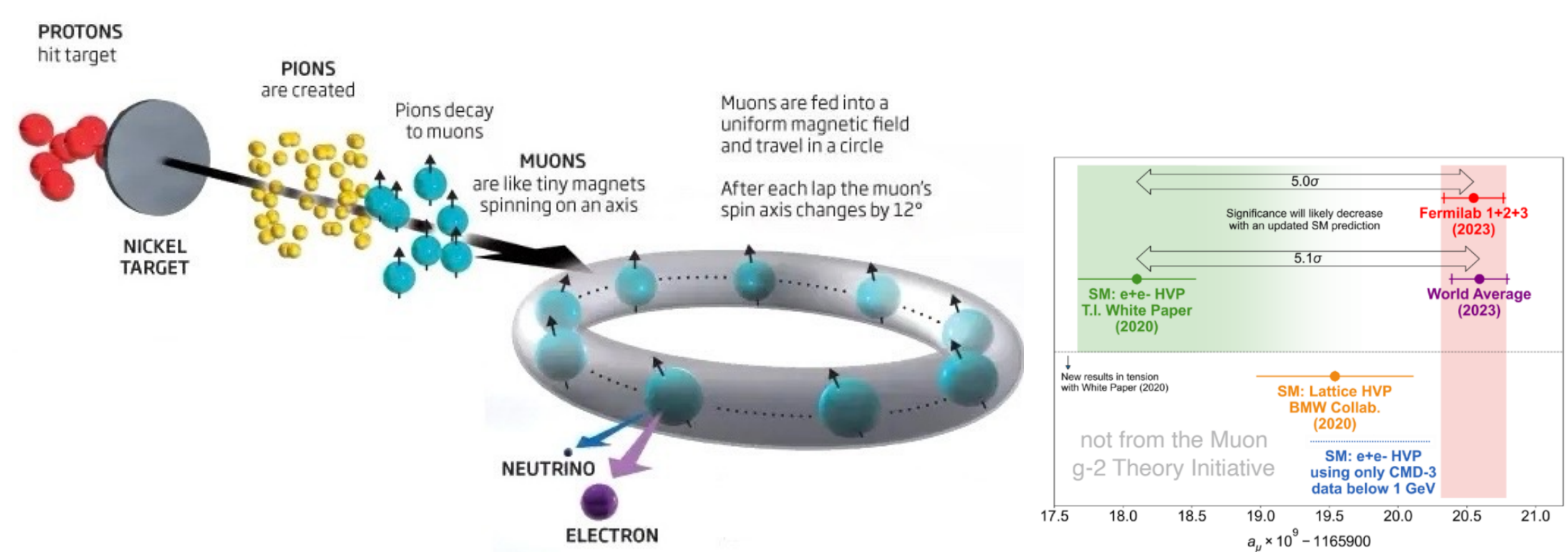


## MOTIVATION: g-2

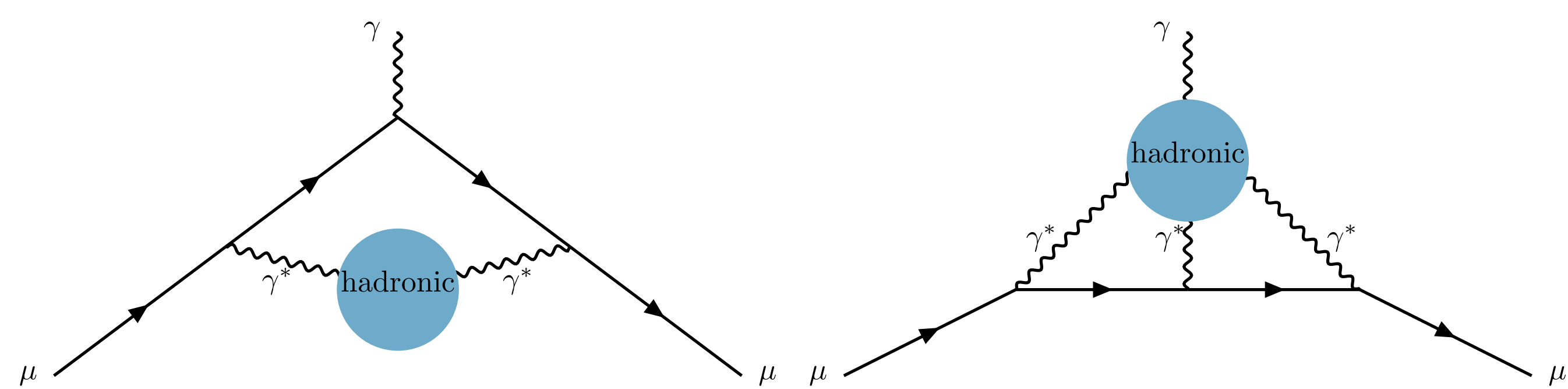
**Anomalous magnetic moment** of the muon: extremely high-precision quantity

- $g$ -factor of muon expected to be 2 from spin:  $\vec{\mu}_s = g \frac{e\vec{s}}{2m}$
- radiative/loop corrections effect deviations:  $\frac{g_\mu - 2}{2} =: a_\mu \neq 0$
- can be measured by observing muon spin precession in a magnetic storage ring
- theory calculation in Standard Model using data-driven dispersive approaches and lattice methods; 2020 value (without lattice):  $a_\mu^{\text{SM}} = 116591810(43) \times 10^{-11}$  [1]
- deviation of  $\sim 5\sigma$  between Ref. [1] and experimental average measurement,  $a_\mu^{\text{exp}} = 116592059(22) \times 10^{-11}$  [2]



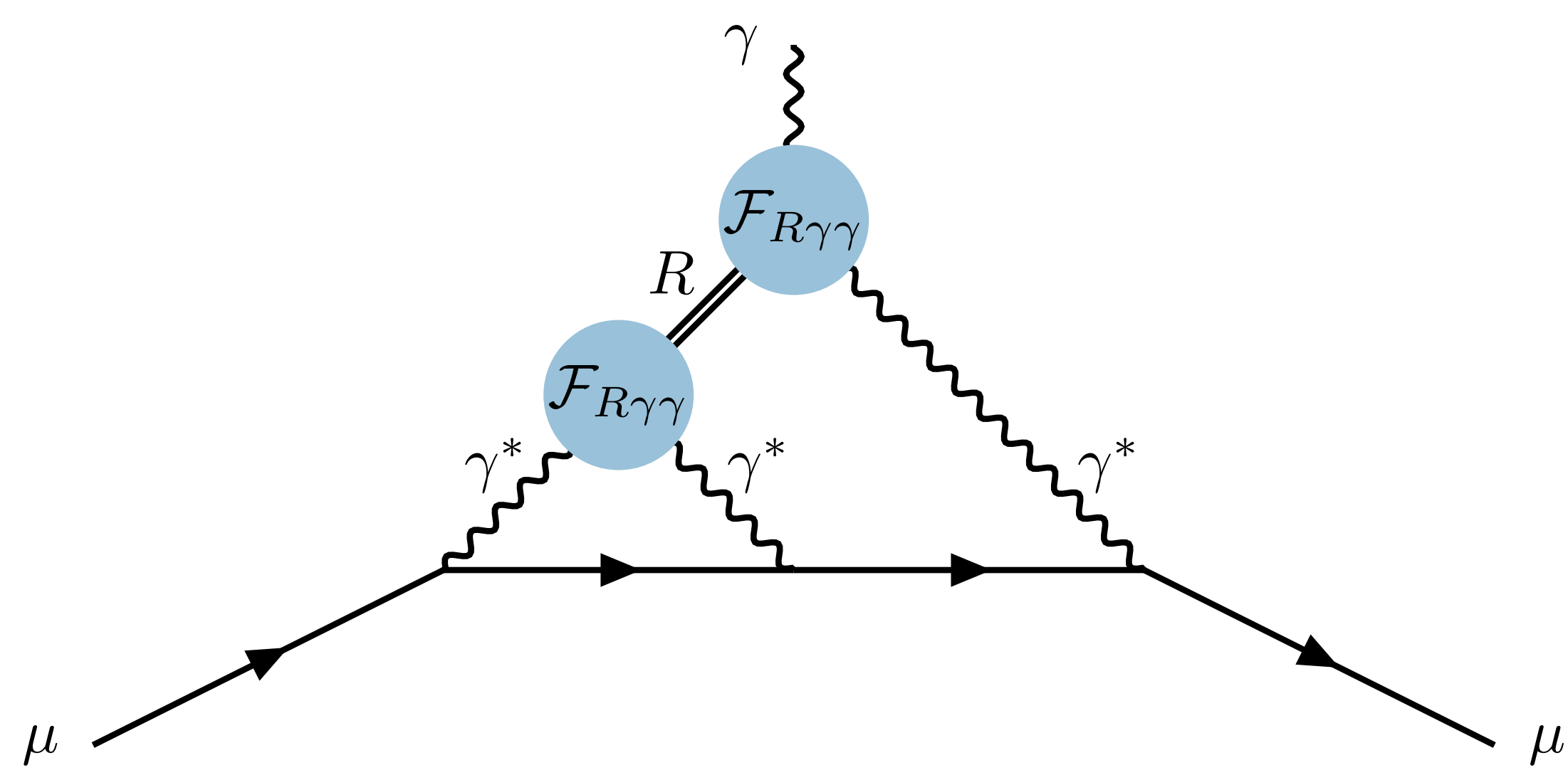
[NewScientist, "Muon whose army?" (19 May 2010)]  
[Bingzhi Li(Unlisted, CN) for the Muon g-2 collaboration. (Apr 29, 2024), PoS HQL2023 (2024) 009]

- $a_\mu$  includes QED and electroweak contributions (well-known)
- currently under debate: hadronic contributions, hadronic vacuum polarization (HVP) and **hadronic light-by-light** scattering (HLbL)



## POLES AND TRANSITION FORM FACTORS

Different intermediate states contribute to the HLbL part of  $a_\mu^{\text{SM}}$ , including resonances  $R$  with quantum numbers  $J^{PC}$



- the resonance couples to (virtual) photons via transition form factor  $\mathcal{F}_{R\gamma\gamma}$
- describe  $\mathcal{F}_{R\gamma\gamma}$  with different models, involving vector-meson dominance and dispersive treatment

## AXIAL-VECTOR & TENSOR MESONS

Can consider an axial-vector ( $A$ ) or a tensor meson ( $T$ ) as the intermediate resonance

**Axial-vector** mesons ( $J^{PC} = 1^{++}$ ) notoriously difficult to measure due to LANDAU-YANG theorem which forbids decay to real photons

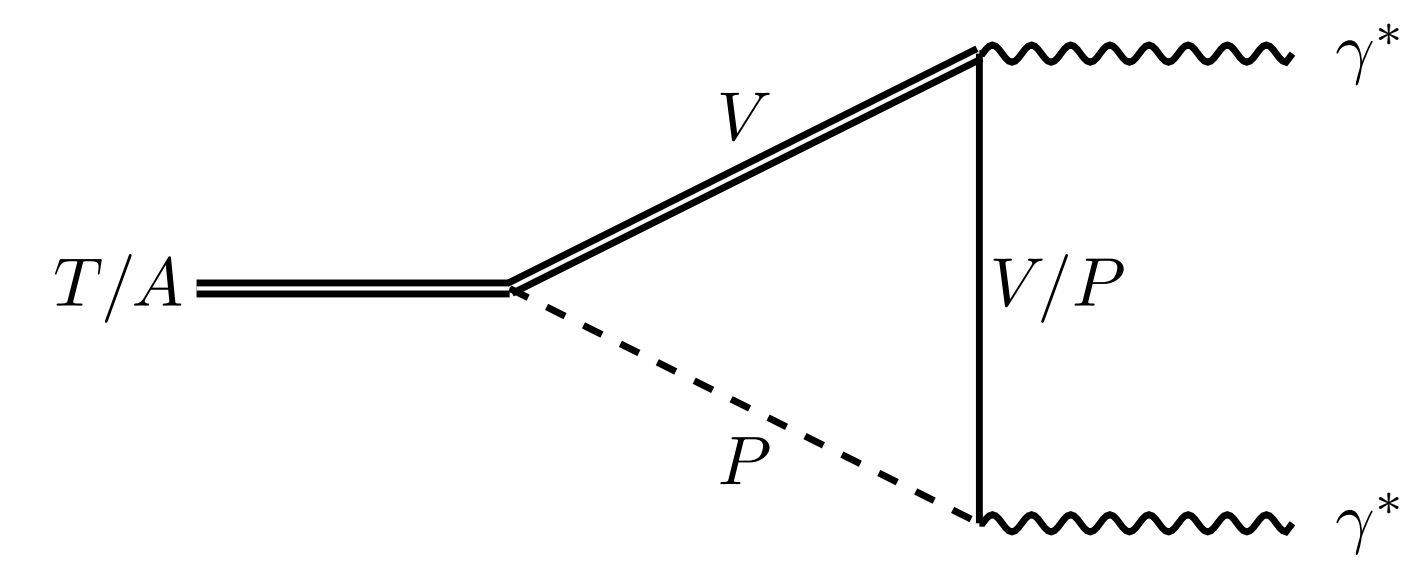
- for  $A = f_1(1285)$ , can make connection to decay  $f_1 \rightarrow e^+e^-$ , scattering  $e^+e^- \rightarrow f_1\pi^+\pi^-$ , and other processes [3, 4] as  $\mathcal{F}_{A\gamma\gamma}$  is universal
- for  $A = a_1(1260)$ , less experimental data, but main decay to  $3\pi$  or  $\rho\pi$

**Tensor mesons** ( $J^{PC} = 2^{++}$ ) are not restricted by the LANDAU-YANG theorem

- for  $T = a_2(1320)$ , the main decay channel is to  $3\pi$  or  $\rho\pi$ ; connection to scattering process  $e^+e^- \rightarrow a_2 e^+e^-$  [5]

## FRAMEWORK

Model the dynamical behavior of  $\mathcal{F}_{A\gamma\gamma}$  and  $\mathcal{F}_{T\gamma\gamma}$  via a left-hand cut

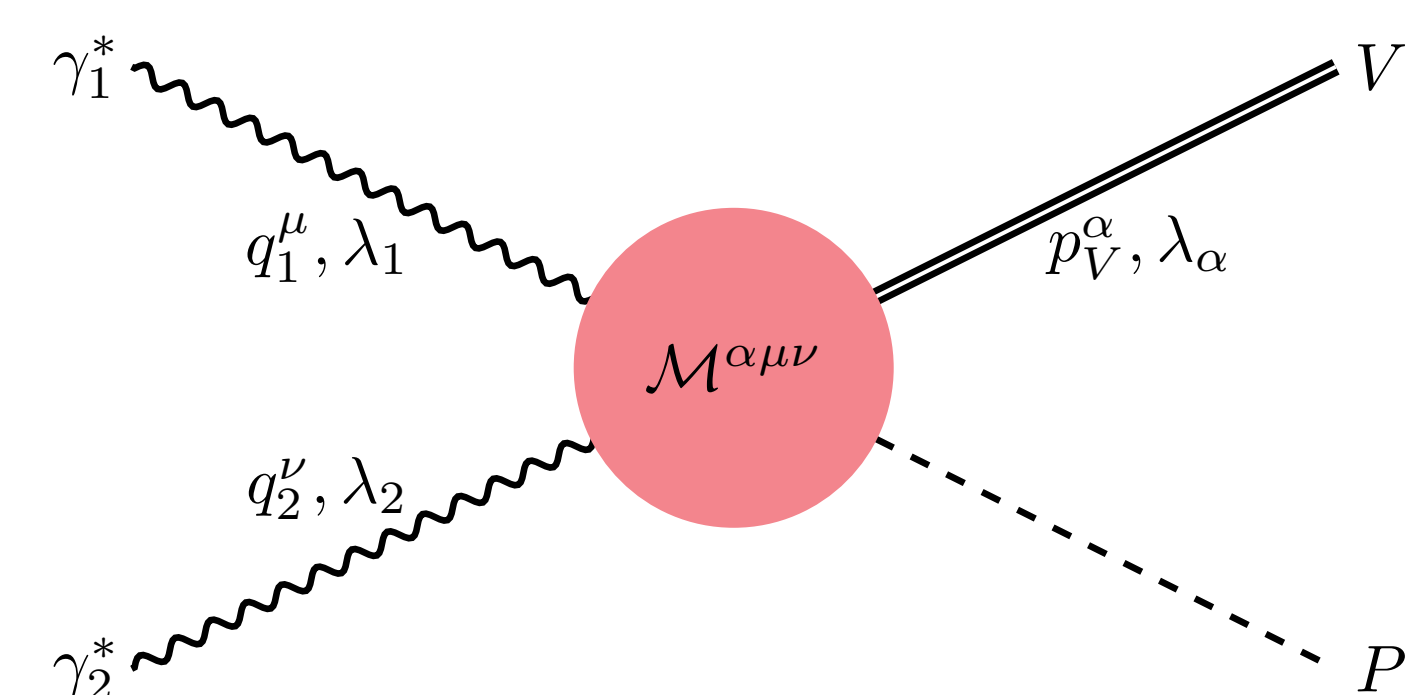


- dispersion relation for left-hand cut, describing the imaginary part in terms of the cross section with a vector-meson/pseudoscalar-meson pole
- framework based on ChPT including field representations for the pseudoscalar (P), vector ( $V_\mu$ ), axial-vector ( $A_\mu$ ) and tensor ( $T_{\mu\nu}$ ) multiplets [6] (containing only the relevant degrees of freedom for considering  $a_1^0/a_2^0 \rightarrow \rho^\pm\pi^\mp$ )

$$P = \sqrt{2} \begin{pmatrix} 0 & \pi^+ & 0 \\ \pi^- & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad V_\mu = \sqrt{2} \begin{pmatrix} 0 & \rho_\mu^+ & 0 \\ \rho_\mu^- & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_\mu = \begin{pmatrix} a_{1\mu}^0 & 0 & 0 \\ 0 & -a_{1\mu}^0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T_{\mu\nu} = \begin{pmatrix} a_{2\mu\nu}^0 & 0 & 0 \\ 0 & -a_{2\mu\nu}^0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

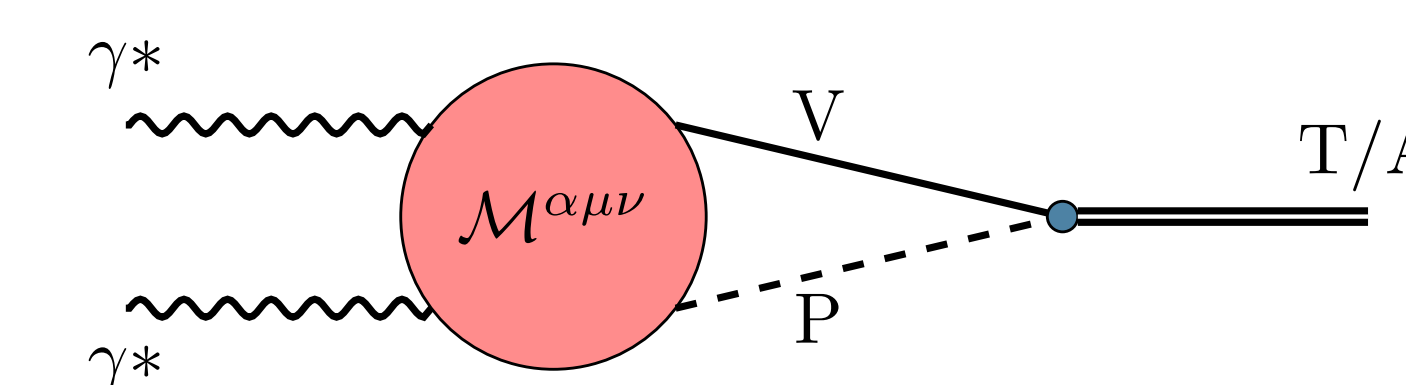
Connect to  $VP \rightarrow \gamma^*\gamma^*$  amplitude  $\mathcal{M}^{\alpha\mu\nu}$  (free indices will be contracted with polarization vectors of  $V, \gamma^*$ )



- decomposition as  $\mathcal{M}^{\alpha\mu\nu} = \sum_i \mathcal{F}_i T_i^{\alpha\mu\nu}$
- need gauge-invariant description of the  $VP \rightarrow \gamma^*\gamma^*$  vertex
- LORENTZ structures  $T_i^{\alpha\mu\nu}$  should be free of kinematic singularities and zeros  $\rightarrow$  form factors  $\mathcal{F}_i$  free of kinematic zeros and singularities (for dispersion relation)
- BTT procedure [7–9]: project  $T_i^{\alpha\mu\nu}$  using WARD identities, form linear combinations in order to remove poles
- use SCHOUTEN identities to find basis for  $\{T_i^{\alpha\mu\nu}\}$

## NEXT STEPS

- contract  $VP \rightarrow \gamma^*\gamma^*$  amplitude with the corresponding vertex for axial-vector or tensor meson,



- identify the form of the remaining form factors, based on dispersion relations, and perform the loop integral
- cross-check the model against experimental data

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