Meshing Around: A Study of Unstructured Spherical Voronoi Tessellation Mesh Grids For Better High-Resolution Numerical Weather Prediction

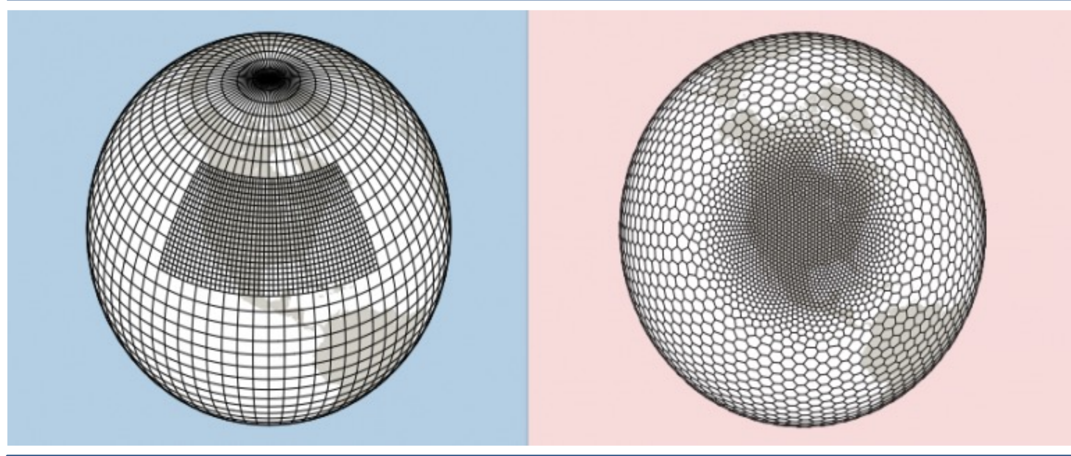
Viktor Loh
Intern, Department of Weather Research, Centre for Climate Research Singapore

METEOROLOGICAL SERVICE SINGAPORE

Motivation

Since their advent in the 1950s, Numerical Weather Prediction (NWP) models have used structured latitude-longitude mesh grids for forecasting, which subdivide the 2D (horizontal) global surface into square grid cells of equal size. This has historically resulted in issues such as unnecessarily high resolution at the poles (the 'pole problem'), and flow distortions at nest boundaries with the use of regional nested models that involve abrupt mesh transitions between coarse and fine domains.

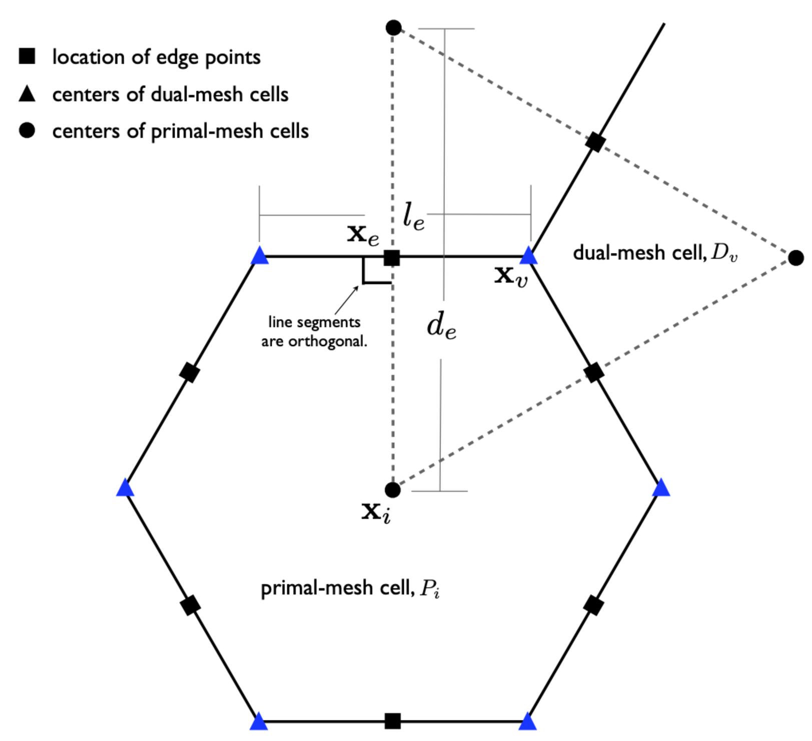
Semi-structured mesh grids have since been explored, but are limited to global meshes of approximately uniform resolution. For variable-resolution Numerical Weather Prediction, an unstructured discretization needs to be adopted.



Introduction

This has led to the development of the Model for Prediction Across Scales (MPAS). MPAS uses a locally orthogonal unstructured discretization; with 5-, 6- and 7-sided polygons of variable cell size and geometry, MPAS mesh grids are a type of staggered Arakawa C-grid, with the primal grid being a Voronoi tessellation and the dual grid being a Delaunay triangulation. By solving for normal velocities on cell edges and conserved quantities at the centroids, these grids scale advantageously on massively parallel computing systems and allow for higher forecast skill - particularly along grid boundaries which show no signs of LBC errors when MPAS is deployed as a regional circulation model (Skamarock et. al., 2018).

The Arakawa C-grid scheme is also desirable in that it conserves mass, potential vorticity and enstrophy, and preserves geostrophic balance. However, using such a scheme requires (besides local orthogonality) that the grid be well-centered and mutually centroidal, constraining the geometry.



The Centre for Climate Research Singapore (CCRS) has recently entered into collaboration with NCAR to explore the development of a next-generation modeling system based on MPAS. Part of this effort involves the generation of unstructured grids over the Maritime Continent region, a term generally referring to the archipelagos of Indonesia, Borneo, New Guinea, the Philippine Islands, the Malay Peninsula, and the surrounding seas. (BoM)

For this work, the Maritime Continent focus region was defined as a circle of radius approximately 1120km centered on the point 1.21175°N, 102.00425°E.

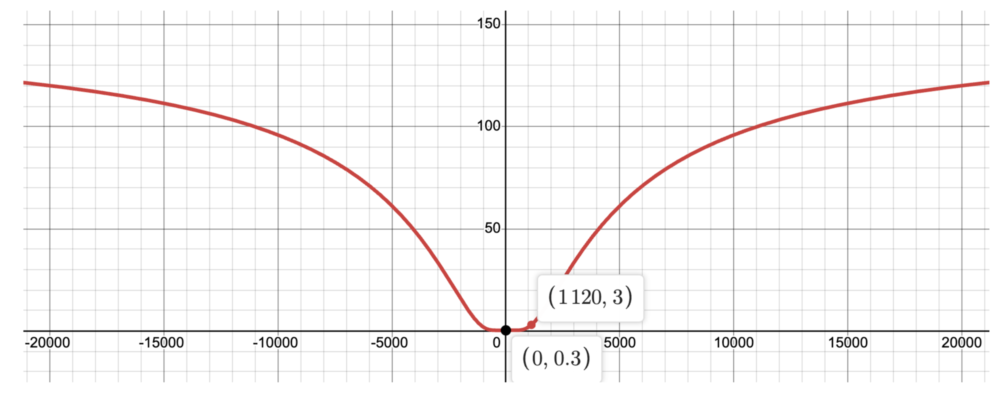
Methodology

These grids can be generated using Lloyd-type algorithms to create *centroidal* SCVT meshes, such as via the MPI-SCVT algorithm (Jacobsen et al., 2013). However, as discussed in Engwirda (2017), the algorithm cannot provide theoretical guarantees on minimum element quality, and is computationally intensive, limiting its applicability to high-resolution use cases. Instead, I explore the use of Engwirda's JIGSAW package for high-resolution mesh generation, which uses a restricted frontal Delaunay refinement algorithm based on an off-center vertex insertion technique. The grids were generated with a user-defined density function (classed as a "bump" function):

$$\bar{h}(x) = r + ae^{-\frac{1}{\frac{|x|}{b}}}$$

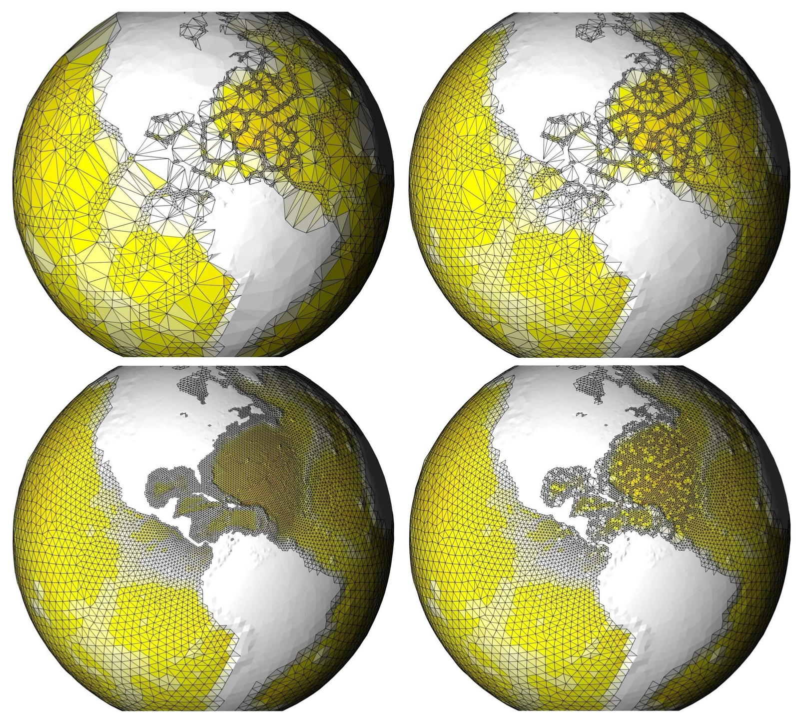
where r is the base resolution of the high-resolution refinement region, and a and b are scaling factors which control the function steepness and radius of the refinement region respectively. The scaling factors chosen were a = 150 and b = 4500, in order to optimize for a base resolution r of 300m leading to a resolution of 3km near the boundary of the focus region (roughly 1120km in radius).

This density function is relatively computationally efficient compared to other Gaussian-type functions that were trialed; these functions were preferred due to their smoothness and applicability as mollifiers, which would minimize the computational effort required for the optimization algorithm to reach convergence.



Generation

The "hill-climbing" mesh optimization algorithm ensures that global mesh quality can only ever increase or remain the same, which makes JIGSAW stand out compared to CVT-type mesh generation approaches. Between refinement and optimization iterations, the new JIGSAW "TETRIS" bisection algorithm is also utilized, which splits each Delaunay triangle into 4 sub-triangles in a manner that preserves the quality of the current geometry. (Engwirda, 2017).



In practice, the greater the number of TETRIS bisections, the greater the proportion of hexagons in the resulting mesh;

TETRIS (N = 0): 90.0% hexagons
TETRIS (N = 1): 95.9% hexagons
TETRIS (N = 2): 97.3% hexagons
TETRIS (N = 3): 97.4% hexagons

These results are observed for a simple 33-150km variable-resolution mesh. However, as the number of bisections increases, the grid becomes too structured and can no longer change resolution effectively; implying an ideal number of bisections for every mesh which maximizes the number of hexagons while minimizing any discontinuities between local element clusters, producing a "best mesh".

The number of bisections N is determined by the function:

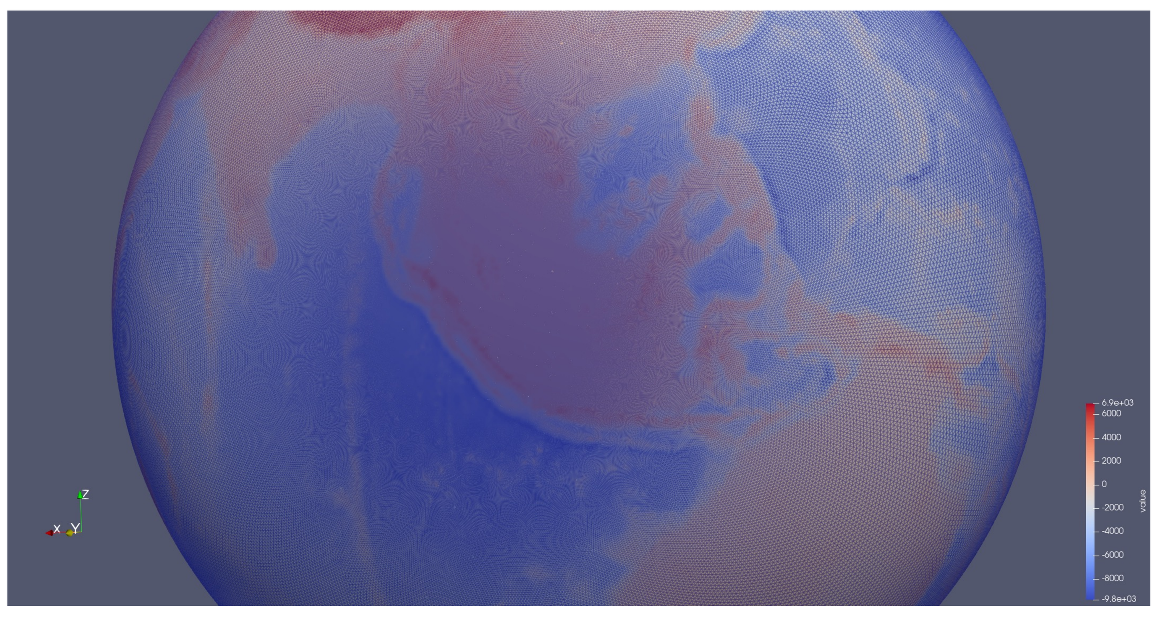
$$N \approx log_2\left(\frac{\frac{\bar{r}}{\sin(0.4\pi)}}{\bar{h}}\right)$$

where $\bar{r}$ is the mean radius of the global ellipsoid and $\bar{h}$ is the mean resolution value of the density function. For a 3km mesh, this returns a value of 7, while for a 300m mesh this returns a value of 11. Further work can be done to explore whether this adequately approximates the ideal number of bisections.

Discussion

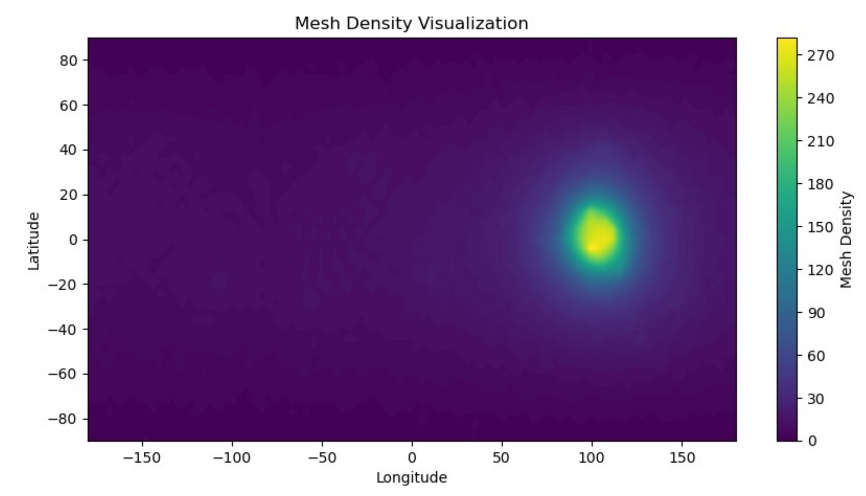
A python script is first used to call the jigsaw-geo-python package, which generates the mesh based on the parameters for the "bump" density function provided. The corresponding jigsaw .msh output file is converted into a raw NetCDF file format using the jigsaw_to_netcdf tool from the maps-tools package, which is then further converted into a MPAS-compatible NetCDF file using the write_netcdf tool by calculating and filling auxiliary fields based on the key geometries defined in the raw file. A graph.info file is also generated to allow for partitioning and running jobs on the mesh in parallel. At the same time, .vtk files for visualization of the mesh in Paraview as well as histogram plots of mesh density are generated.

To validate the meshes, the MPAS-Atmosphere model is then used to generate static fields and initialized conditions for the meshes. If static fields and initialized conditions can be successfully generated for a mesh without broken fields, the mesh has been successfully generated and is usable in principle. .static and .init model files are then run through the convert_mpas program to interpolate the unstructured grid fields onto a latitude-longitude grid, allowing plots to be visualized using the NCAR Command Language (NCL).



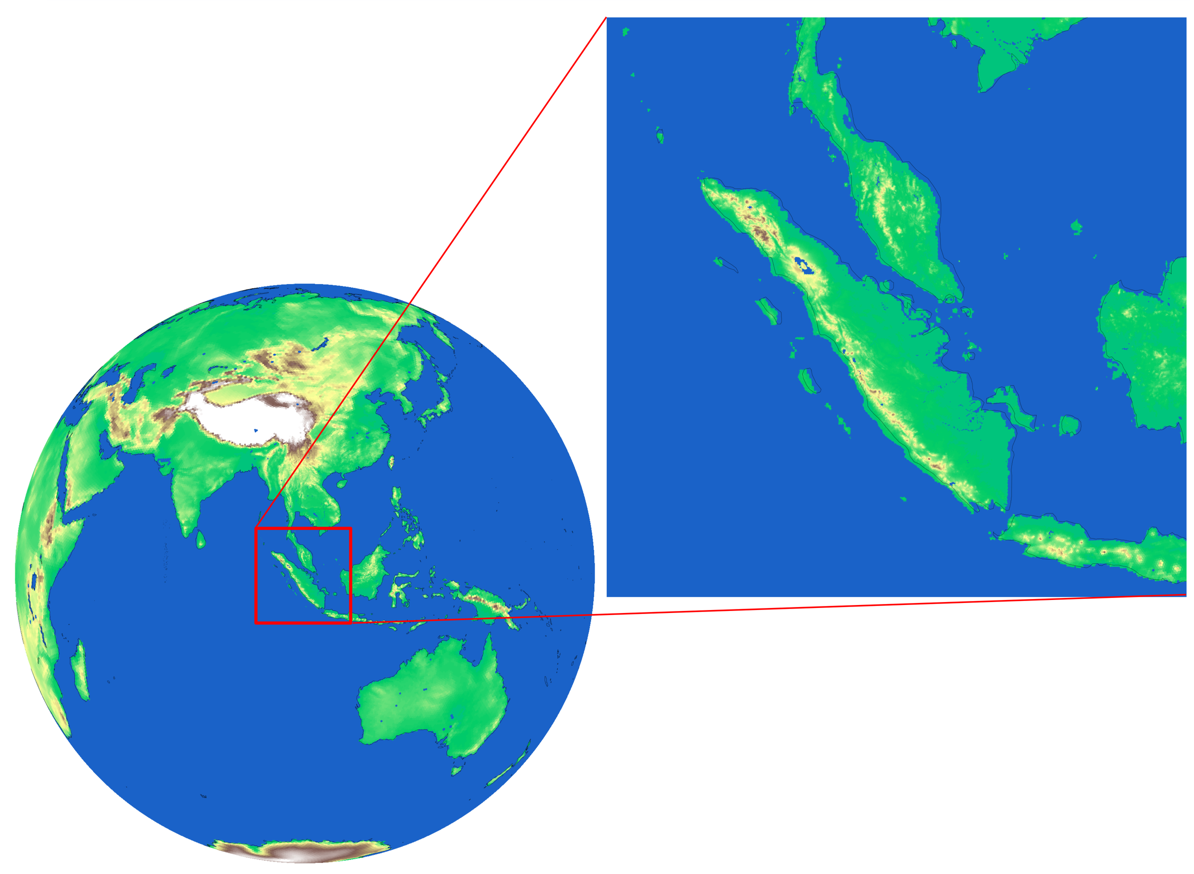
Conclusions

300m, 500m, 1km and 3km meshes were generated for the Maritime Continent focus region. 3D plots shown were rendered with Paraview and the mesh density histograms were plotted with matplotlib.



As can be seen, largely well-structured meshes were successfully generated, with minimal irregularity arising from pentagonal cells in the mesh as a sort of very weak discontinuity between neighboring regions of highly ordered hexagonal cells. Such artifacts in the unstructured mesh should not have any observable effect on the quality of the model and are similarly produced in high-resolution use-cases of other algorithms such as those applying a Centroidal (CVT) approach (M. Duda, personal communication, July 11th, 2023). Given the use of aggressively non-uniform JIGSAW grids (such as those with coastal and bathymetric refinement) for the MPAS-Ocean model, we do not expect any impact to model performance caused by our grids.

An example of terrain fields for the focus region successfully interpolated onto the unstructured mesh is provided as proof-of-concept.



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Further Information

Email Viktor at viktor.loh@columbia.edu