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On the Asymptotic Persistence of Langmuir Modes in Kinematically Complex Plasma Flows

Ketevan Arabuli

Supervisor: Prof. Stefaan Poedts
Co-supervisor: Prof. Andria Rogava

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Introduction to Shear Flow

- **Definition:** Shear flow refers to fluid motion where adjacent layers move at different velocities.
- **Characteristics:** Non-uniform velocity profiles.
- **In Hydrodynamics:** Linked with solid boundaries and fluid viscosity.
- **In Astrophysics:** Influenced by intrinsic inhomogeneity of gravitational and electromagnetic interactions.

Shear Flows in Nature

- **Planetary Flows**
  Mixing of ocean layers, atmospheric jet streams in terrestrial planets, zonal jets in Gas giants.
- **Solar Flows**
  Solar differential rotation, generation of the solar dynamo.
- **Accretion flows**
  Angular momentum transport, magnetic stability of the disk.
- **Galactic flows**
  Structural features of emerging galaxies, star formation efficiency, alignment in the cosmic web.

Historical Background

Hydrodynamical Shear Flows

- Hagen-Poiseuille flow
- Plane Couette flow
- Turbulent shear flows

Astrophysical Shear Flows

- Galactic density waves
- Nonaxisymmetric shear perturbations in accretion disks
- Generation of the solar MHD waves, the solar wind acceleration
- Galactic and extragalactic jets

Methodology

Classical Theory Limitations:

- Fails to match relevant experimental results.
- The issue is caused by the non-self-adjoint behavior of governing equations.

Nonmodal Approach:

- Originates from Kelvin's work.
- Handles non-exponential disturbance behavior.
- Converts PDEs to ODEs.

Method developed by (Mahajan and Rogava, 1999)

Nonmodal Approach

Only consider small-scale perturbations, with $l_i \leq L_i$. The spatial inhomogeneity of an arbitrary background velocity field $\mathbf{U}(x, y, z)$ in the close neighbourhood of a point $A(x_0, y_0, z_0)$ such that $(|x - x_0|/|x_0| \ll 1)$, can be approximated by the linear terms in its Taylor expansion:

$$\mathbf{S} \equiv \begin{bmatrix} U_{x,x} & U_{x,y} & U_{x,z} \\ U_{y,x} & U_{y,y} & U_{y,} \\ U_{z,x} & U_{z,y} & U_{z,z} \end{bmatrix} \tag{1}$$

The linearized convective derivative reduces to:

$$\mathcal{D}u_i + a_{ik}u_k, \tag{2}$$

where $\mathcal{D} = \partial_t + U_i(x, y, z)\partial_i$ is a spatially inhomogeneous operator.

Nonmodal Approach

For any fluctuation $F(x, y, z; t)$, we consider the ansatz of the form:

$$F(x, y, z; t) \equiv \hat{F}[\mathbf{k}(t), t]e^{i\phi}, \tag{3a}$$

$$\phi(t) \equiv k_i(t)x_i - U_{0i}\int_0^t k_i'(t)dt, \tag{3b}$$

The convective derivative becomes an ordinary derivative in time:

$$\mathcal{D}F = e^{i\phi}\partial_t\hat{F}. \tag{4}$$

But this only holds if the wavevector $\mathbf{k}(t)$ acquires the time dependence given by:

$$\partial_t\mathbf{k} + \mathbf{S}^T \cdot \mathbf{k} = 0, \tag{5}$$

Linear theory of Langmuir modes in kinematically complex shear flows

For the mean flow velocity field $U_{xx} = -U_{yy} \equiv \sigma, U_{xy} \equiv a, U_{yx} \equiv b$. Applying the ansatz, we derive the set of linearized, first-order ODEs:

$$d_\tau \varrho = k_x v_x + k_y v_y, \tag{6a}$$

$$d_\tau v_x = -\epsilon v_x - R_1 v_y - (W/K)^2 k_x \varrho, \tag{6b}$$

$$d_\tau v_y = -R_2 v_x + \epsilon v_y - (W/K)^2 k_y \varrho. \tag{6c}$$

$k_{x,y}$ stands for the dimensionless wavevector components obeying:

$$d_\tau k_x = -\epsilon k_x - R_2 k_y, \tag{7a}$$

$$d_\tau k_y = -R_1 k_x + \epsilon k_y, \tag{7b}$$

Linear theory of Langmuir modes in kinematically complex shear flows

This set of equations involve two conserved quantities $\Delta \equiv d_\tau k_x k_y - d_\tau k_y k_x$ and $\mathcal{C} \equiv k_y v_x - k_x v_y - (R_2 - R_1)\varrho$.

Taking one more time derivative of the wavevector equations, leads to:

$$\begin{Bmatrix} k_x \\ k_y \end{Bmatrix}^{(2)} = \Lambda^2 \begin{Bmatrix} k_x \\ k_y \end{Bmatrix} \tag{8}$$

where $\Lambda^2 \equiv \epsilon^2 + R_1 R_2$.

Linear theory of Langmuir modes in kinematically complex shear flows

The velocity perturbation components in terms of ϱ and K:

$$v_x = \frac{1}{K^2}\left\{k_y\left[\mathcal{C} + (R_2 - R_1)\varrho\right] + k_x\varrho^{(1)}\right\}, \tag{9a}$$

$$v_y = \frac{1}{K^2}\left\{-k_x\left[\mathcal{C} + (R_2 - R_1)\varrho\right] + k_y\varrho^{(1)}\right\}. \tag{9b}$$

From the equation of continuity:

$$\varrho^{(2)} + \left[W^2 - 2\Delta\frac{(R_2 - R_1)}{K^2}\right]\varrho - \frac{(K^2)^{(1)}}{K^2}\varrho^{(1)} = \frac{2\Delta\mathcal{C}}{K^2}. \tag{10}$$

Introducing the auxiliary variable $\Phi \equiv \varrho/K$:

$$\Phi^{(2)} + \left[W^2 - \Lambda^2 + \frac{3\Delta^2}{K^4}\right]\Phi = \frac{2\mathcal{C}\Delta}{K^3}, \tag{11}$$

Possible regimes of Langmuir modes in kinematically complex shear flows

- Conserved quantity $\Delta = 0$

$$\Psi^{(2)} + [W^2 - \Lambda^2]\Psi = 0. \tag{12}$$

- The case $\Lambda^2 = 0$

$$K^2(\tau) = k^2(0) + \partial_\tau k^2(0)\tau + (R_1 - R_2)\Delta\tau^2 \tag{13}$$

- The case $\Lambda^2 > 0$

$$K^2(\tau) = \delta + \mathcal{A}\cosh\left(2\Lambda\tau + \psi_0\right), \tag{14}$$

- The case $\Lambda^2 < 0$

$$K^2(\tau) = \delta + \mathcal{B}\cos\left(2\Omega\tau + \psi_0\right), \tag{15}$$

Possible regimes of Langmuir modes in kinematically complex shear flows

The case $\Lambda^2 > 0$

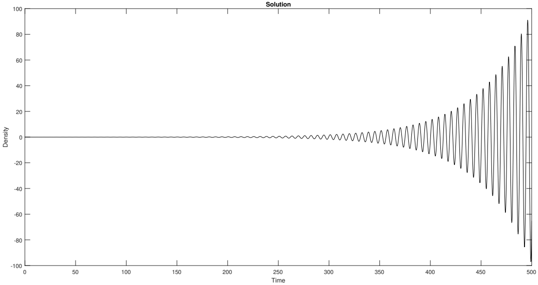

Figure: Exponentially growing density perturbation ϱ, for $R_1 = 2 \cdot 10^{-3}$, $R_2 = 0.2$, $\varepsilon = 0$, $k_y(0) = 0.1$, $C = 0$, $W = 1$, $\Psi(0) = 10^{-2}$, and $\Psi'(0) = 0$.

Possible regimes of Langmuir modes in kinematically complex shear flows

The case $\Lambda^2 < 0$

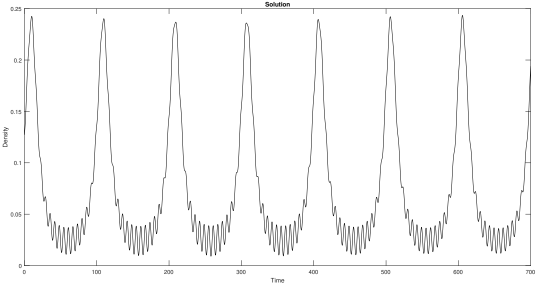

Figure: Asymptotic persistence of echoing SLV interacting with LW for the density ϱ perturbations. $\varepsilon = 0$, $R_1 = 0.1$, $R_2 = -0.01$, $\mathcal{C} = 1.2$, $W = 1$, $\mathcal{K}_y(0) = 1$, $\Psi(0) = 9 \cdot 10^{-2}$, and $\Psi^{'}(0) = 8.66 \cdot 10^{-3}$.

Conclusion and Future prospects

Langmuir Waves in Solar Physics

- **Role in Energy Exchange:** Langmuir waves facilitate energy exchange between particle species in the heliosphere.
- **Solar Radio Bursts:** Type III solar radio bursts are linked to Langmuir waves excited by energetic electrons.
- **Cometary Interactions:**
  - Interaction with solar wind
  - Formation of cometary magnetotails
- **Interstellar Medium:** Voyager 1 detected Langmuir waves beyond the heliopause.

Conclusion and Future prospects

The shear-induced phenomena characterized by asymptotic persistence:

- Exponentially growing solution.
- The echoing solution with persistent wave-vortex-wave conversions.
- The existence of parametrically unstable wave solutions also expected.

Mahajan, S. M. and Rogava, A. D. (1999). What can the kinematic complexity of astrophysical shear flows lead to? The Astrophysical Journal, 518(2):814.

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