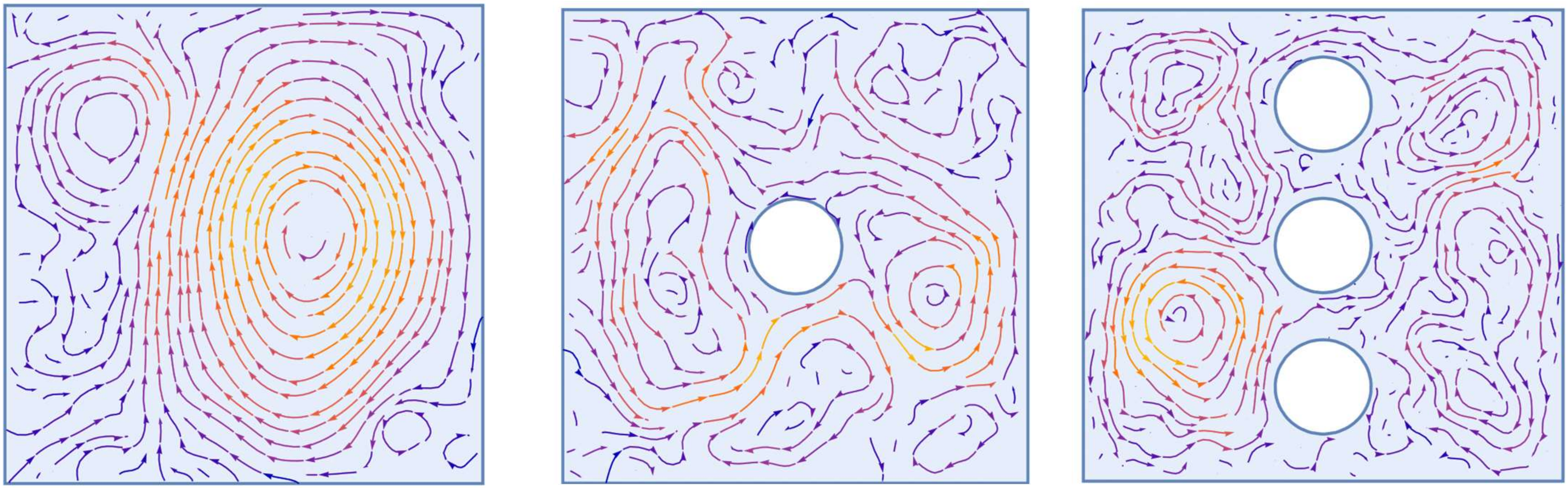


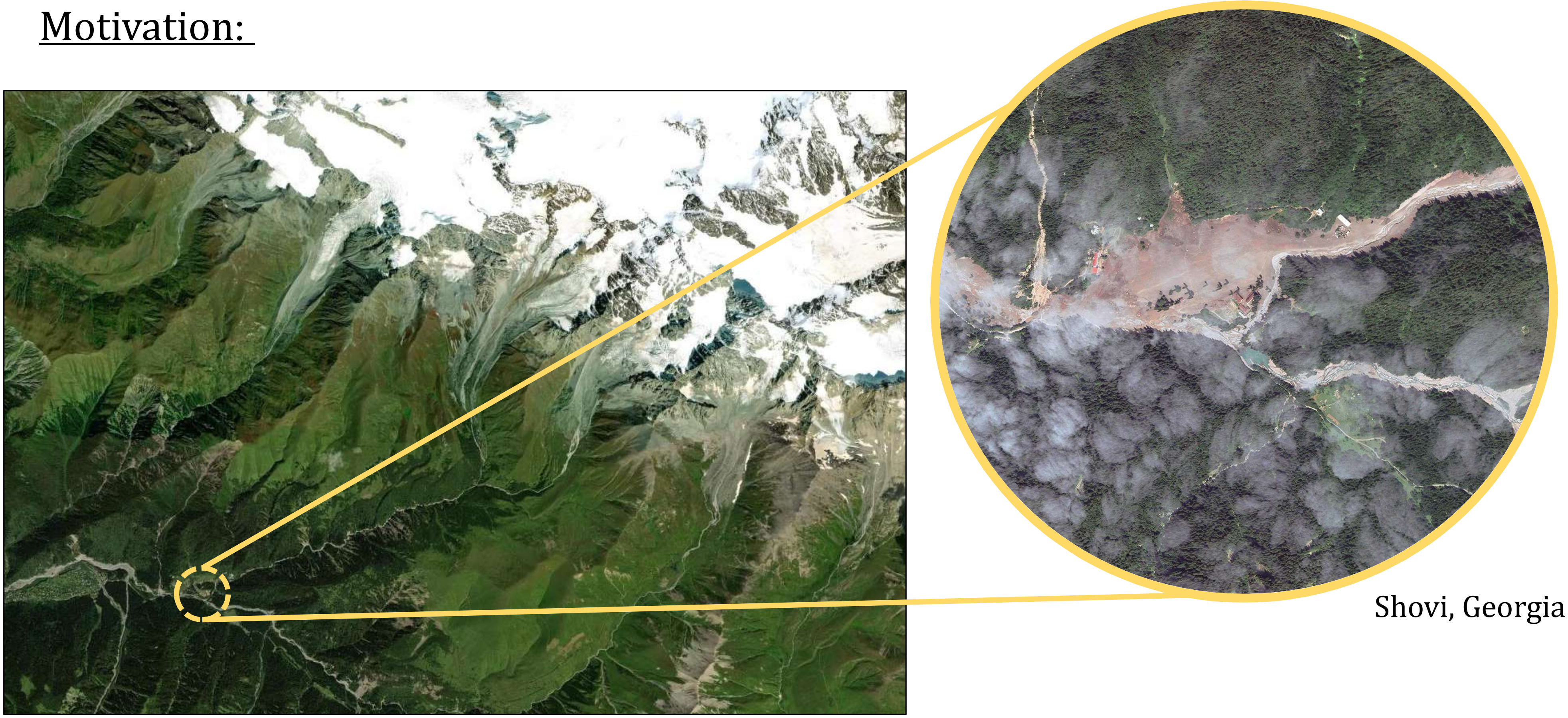
Rheological Equation and application on Terain lattice

REPORTER: Zurabi Ugulava

Outline:

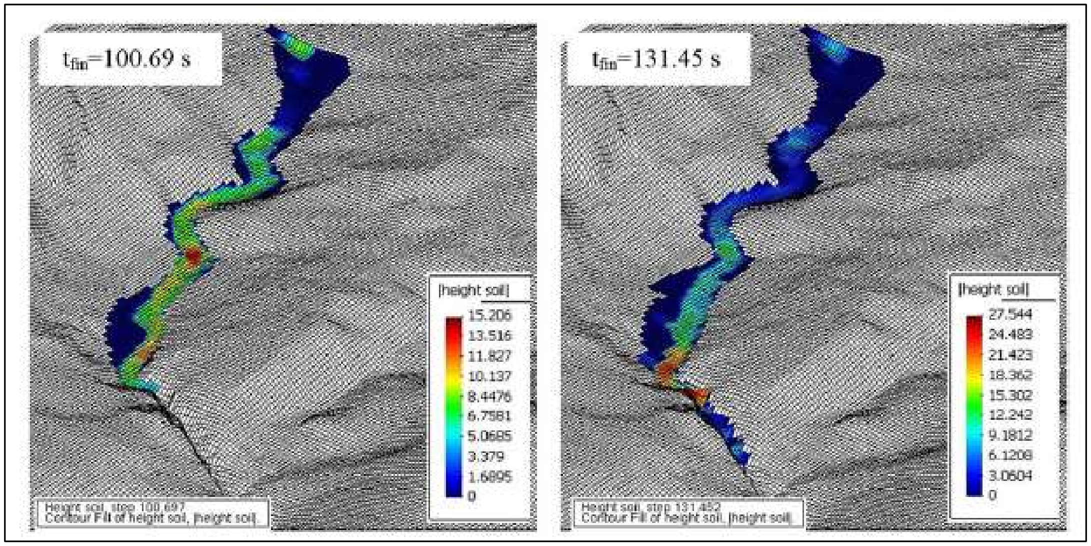
- Motivation
- General Mudflow description
  - Dynamics of flow
  - 1D and 2D cases
- Interaction with bridge pier
- Automatisation mechanism
  - Schematic introduction
  - Surface-mesh analysis and lattice construction
- Results
- Summary

Motivation:

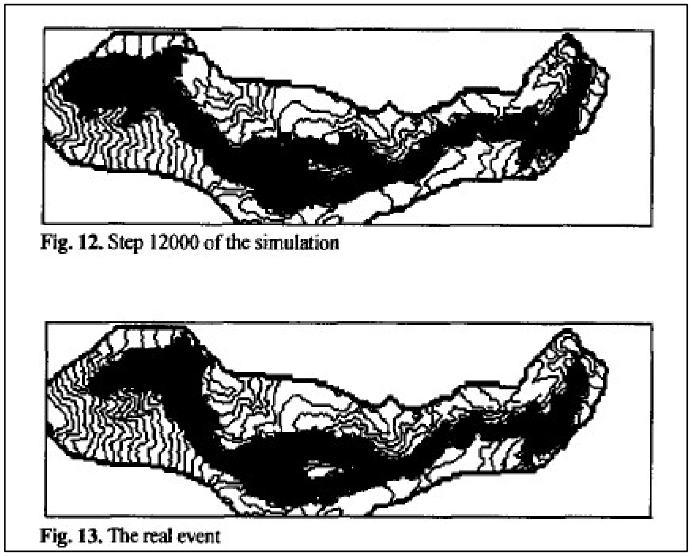


Shovi, Georgia

Approaches:



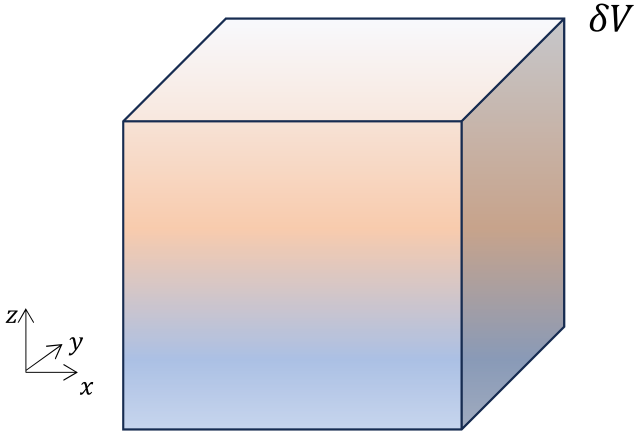
E. Žic et al.: A model of mudflow propagation downstream from the Grohovo landslide near Rijeka



S.Di Gregorio- Mount Ontake Landslide Simulation by Cellular Automata Model SCIDDICA-3

1. General Fluid Description
2. Special scenarios

General Fluid description:



infinitestmal element of the fluid

s_0 – water concentration

ρ_0 – water density

mass of infinitesmal element:

$$\delta m = \rho \delta V = \sum_i \boxed{\rho_i} \delta V_i = \sum_i \rho_i \boxed{s_i} \delta V$$

Component density — concentration level

Effective density:

$$\rho = s_i \rho^i$$

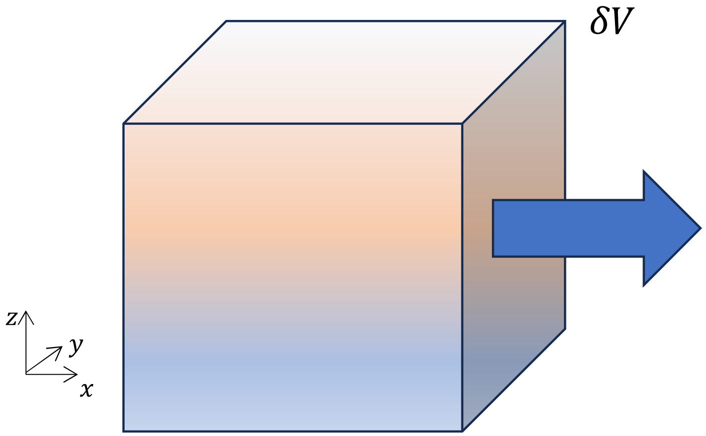
Constrain:

$$\sum_{n-1} s_i = s_1 + \cdots + s_{n-1} = 1 - s_0$$

Two component mixture:

$$\rho_{II} = s\rho_0 + (1-s)\rho_1 = \rho_1 + (\rho_0 - \rho_1)s$$

General Fluid description:



infinitestmal element of the fluid

s_0 – water concentration

ρ_0 – water density

Tensor notation for Navier – Stokes equations:

$$\frac{\partial}{\partial t}(\rho v_i) + \Pi_{ik,k} = 0 \qquad \Pi_{ik} = \sigma_{ik} + \rho v_i v_k$$

Geniev-Gogoladze Stress Tensor:

$$\sigma_{ik} = -p\delta_{ik} + (\mu + \lambda p)\left(v_{i,k} + v_{k,i}\right)$$

Parametrization:

$$\lambda(s_1, \ldots, s_n) = \lambda(0, \ldots, 0) + \mathrm{grad}\,\lambda \cdot \vec{s} + \mathcal{O}(s^2)$$

System of Equations:

Continuity Equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x^k} v_k + \rho v_{i,i} = 0$$

Navier – Stokes Equations

Function of $\vec{s}$

$$\rho\left(\frac{\partial v_i}{\partial t} + \frac{\partial}{\partial x^k}(v_i v_k)\right) = -\sigma_{ik,k} + \phi_i$$

set of Diffusion equations:

$$\frac{\partial s_i}{\partial t} = D_i \nabla^2 s_i$$

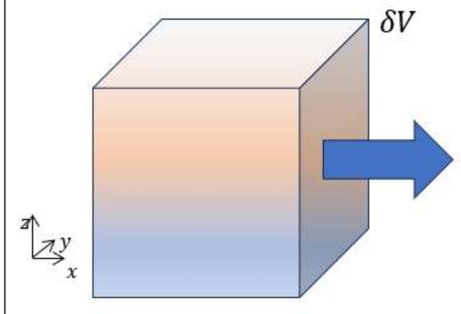
Boundary conditions:

$\{x, y, z\} \in \partial\Omega$ — $v = 0$

$\{x, y, z\} \in \partial\Omega_1$ — $p = p_0$, $s_0 = 1$

$\partial\Omega$ – full boundary

$\partial\Omega_1$ – free surface boundary

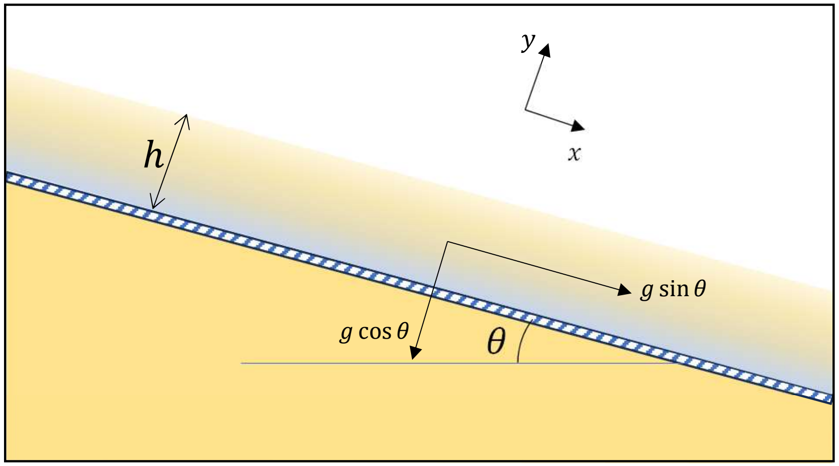


infinitestmal element of the fluid

1 General Fluid Description

2 Special scenarios: 1D flow

Stabilized flow on inclined surface:



$$t \to \infty \qquad \frac{\partial s}{\partial t} = \frac{\partial \vec{v}}{\partial t} = 0$$

No further diffusion present:

$$\frac{\partial s_i}{\partial t} = D_i \nabla^2 s_i = 0$$

X – axis:

$$\frac{1}{\rho}\frac{\partial v}{\partial y}\left(p\frac{\partial \lambda}{\partial y} + \lambda\frac{\partial p}{\partial y}\right) + (\mu + \lambda p)\frac{1}{\rho}\frac{\partial^2 v}{\partial y^2} + g\sin\theta = 0$$

Y – axis:

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} + g\cos\theta = 0$$

Sediment distribudion: Constrain:

$$s_i(y) = k_i y + b_i$$

$s(h) = 0$

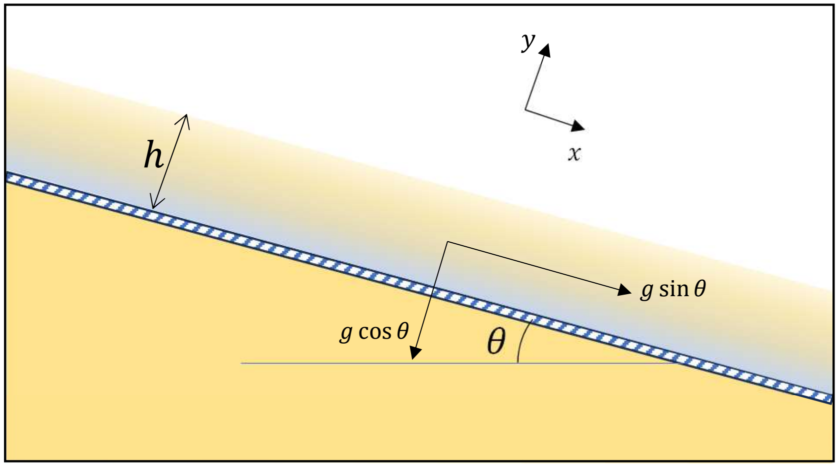
$s(0) = 1$

$$s_i(y) = 1 - \frac{y}{h}$$

density distribution:

$$\rho(y) = s_i\rho_i + s_0\rho_0 = \rho_i + (\rho_w - \rho_i)\frac{y}{h}$$

Stabilized flow on inclined surface:



Sediment distribudion: Constrain:

$$s_i(y) = ky + b \quad s(h) = 0,\ s(0) = 1 \quad s_i(y) = 1 - \frac{y}{h}$$

density distribudion:

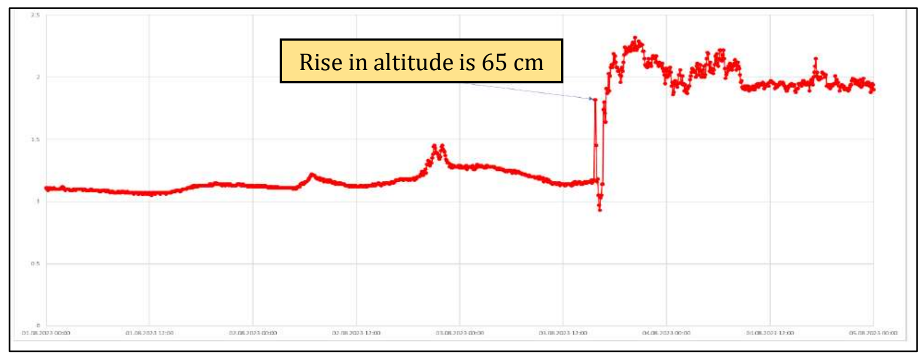
$$\rho(y) = s_i\rho_i + s_0\rho_0 = \rho_i + (\rho_w - \rho_i)\frac{y}{h}$$

Y – axis:

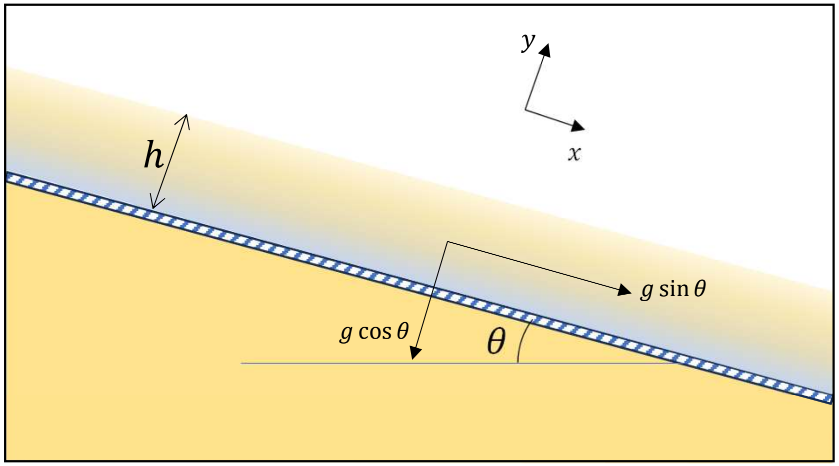
$$\frac{1}{\rho}\frac{\partial p}{\partial y} + g\cos\theta = 0$$

$$p = Ay^2 + By + P_0$$

$$A = \frac{1}{2} g\cos\theta \sum_i \rho_i \qquad B = \frac{1}{2} g\cos\theta \sum_i b_i$$



Stabilized flow on inclined surface:



Sediment distribudion:

Constrain:

$$s_i(y) = ky + b \quad s(h) = 0,\ s(0) = 1 \quad s_i(y) = 1 - \frac{y}{h}$$

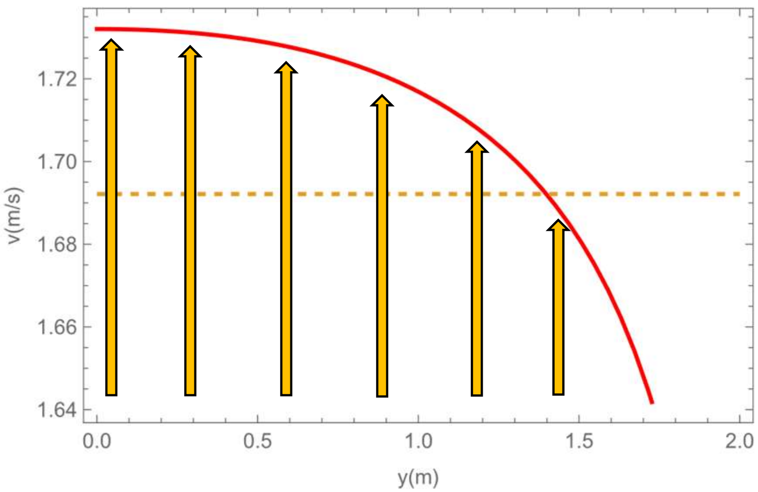
density distribudion:

$$\rho(y) = s_i\rho_i + s_0\rho_0 = \rho_i + (\rho_w - \rho_i)\frac{y}{h}$$

X – axis:

$$\frac{1}{\rho}\frac{\partial v}{\partial y}\left(p\frac{\partial \lambda}{\partial y} + \lambda\frac{\partial p}{\partial y}\right) + (\mu + \lambda p)\frac{1}{\rho}\frac{\partial^2 v}{\partial y^2} + g\sin\theta = 0$$

$$p = -\left(\rho_i + \frac{y}{2h}\delta\rho\right) gy\cos\theta + P_0$$



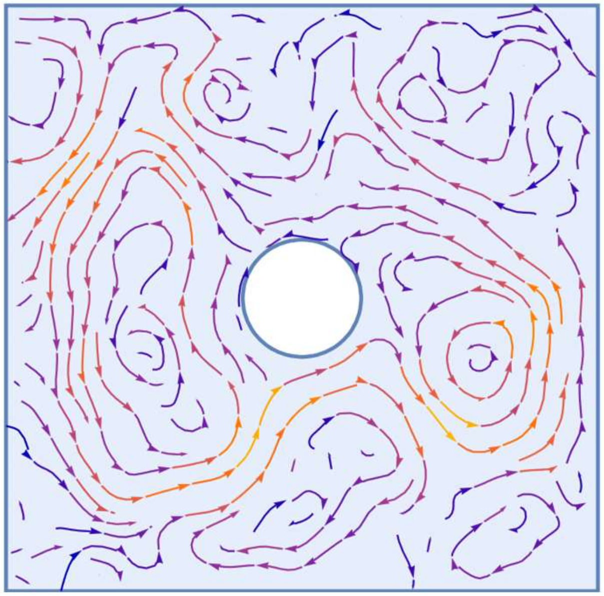
Mean profile velocity:

$$\bar{v} = \frac{1}{h}\int v(y)dy$$

1 General Fluid Description

2 Special scenarios: 2D flow

Stabilized flow on 2D surface around Pier:



$$t \to \infty \qquad \frac{\partial s}{\partial t} = \frac{\partial \vec{v}}{\partial t} = 0$$

Continuity Equation:

$$\frac{\partial \rho}{\partial x^k} v_k + \rho v_{i,i} = 0$$

set of Diffusion equations:

$$D_i \nabla^2 s_i = 0$$

Navier – Stokes Equations

$$\rho \left(\frac{\partial v_i}{\partial t} + \frac{\partial}{\partial x^k} (v_i v_k) \right) = -\sigma_{ik,k} + \phi_i$$

Boundary conditions:

$\{x, y, z\} \in \partial\Omega$ —— $\vec{v} = 0$

$\partial\Omega'$

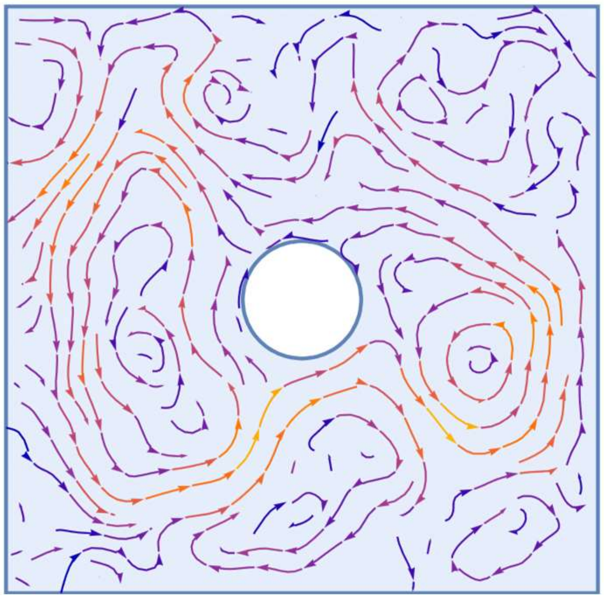
$x = 0$ → $p = 0$

$x = 0$ → $v_x = const$

$\partial\Omega$ – full boundary

$\partial\Omega_1$ – free surface boundary

Stabilized flow on 2D surface around Pier:



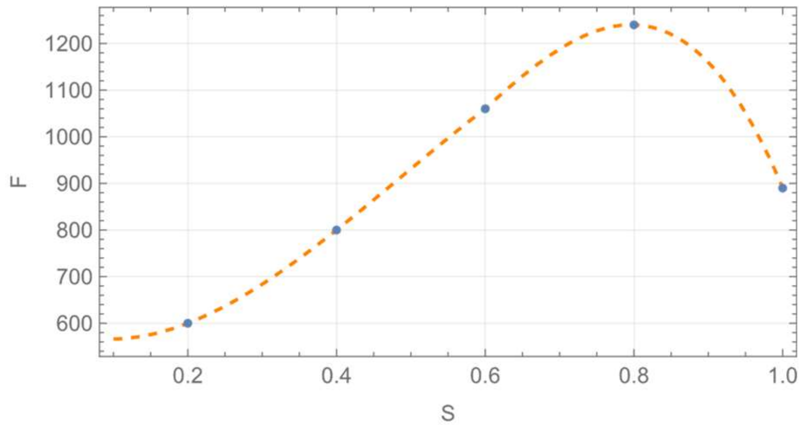
Force acting on the pier:

$$F = \oiint \sigma_{ik}\hat{n}_k \cdot d\vec{r}$$

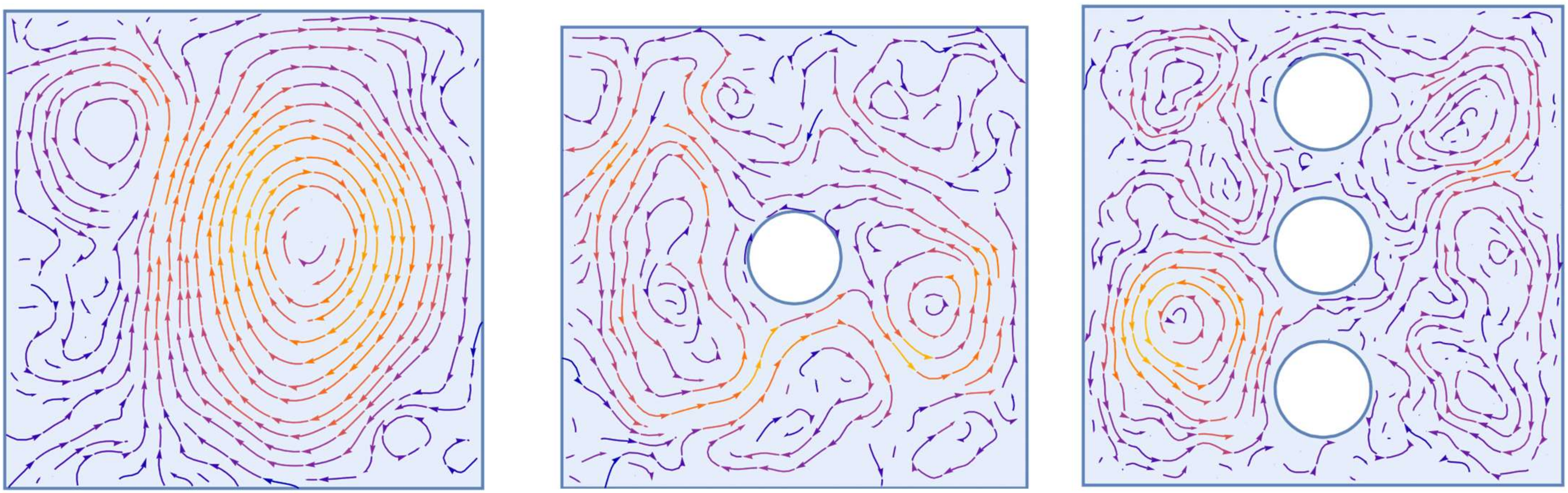
Geniev-Gogoladze Stress Tensor:

$$\sigma_{ik} = -p\delta_{ik} + (\mu + \lambda p)\left(v_{i,k} + v_{k,i}\right)$$

$$F = -\oiint p\hat{n}_k \cdot d\vec{r} + \oiint (\mu + \lambda p)\left(v_{i,k} + v_{k,i}\right)\hat{n}_k \cdot d\vec{r}$$



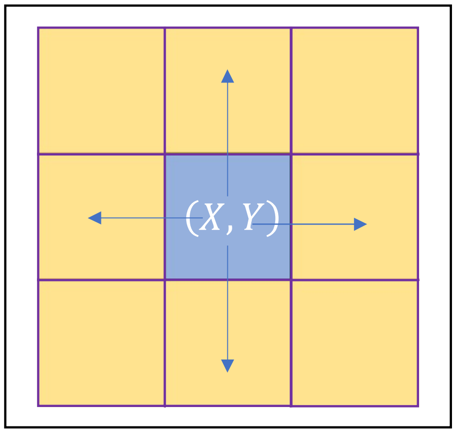
Increasing Column Count:



① General Fluid Description

② statistical model

Automatization mechanism:



State distribution:

$$p_{mn} \sim \rho(X,Y) s(X,Y)\, \mathcal{F}(X,Y)\, h_{m,n}(X,Y)$$

density dist

altitude dist

concentration dist

$$p_{mn} \sim \rho(X,Y) s(X,Y)\, \mathcal{F}(X,Y)\, h_{m,n}(X,Y)$$

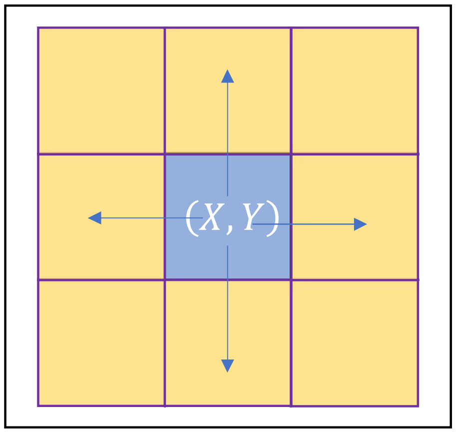
$$\delta h_{mn} = h_m - h_n$$

$$\delta h_{mn} < 0 \mid h_{m,n} = 0$$

$$\delta h_{mn} > 0 \mid \quad h_n = h_n + \frac{\rho v^2}{2\delta h_{mn} g}(1 - f)$$

$$\mid \quad h_m = h_m - \frac{\rho v^2}{2\delta h_{mn} g}(1 - f)$$

Automatization mechanism:



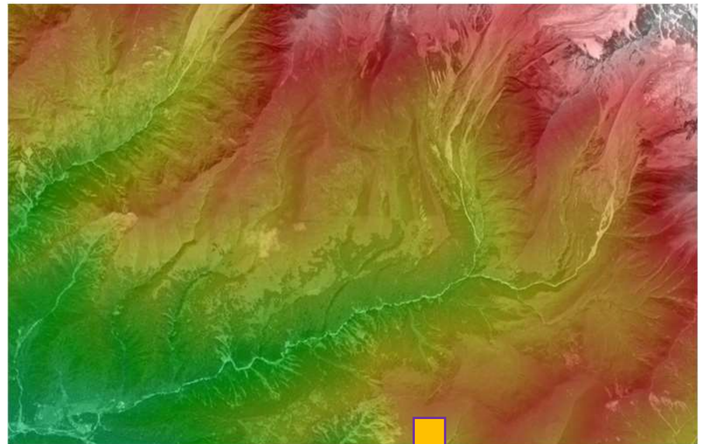
State distribution:

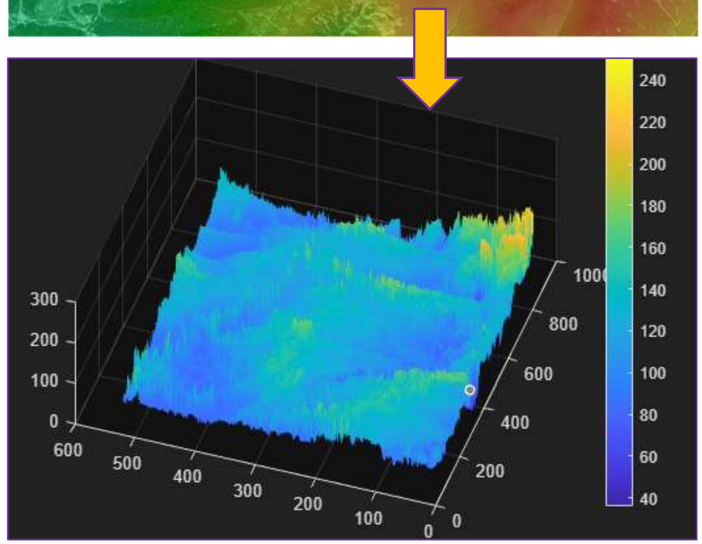
$$p_{mn} \sim \rho(X,Y) s(X,Y)\, \mathcal{F}(X,Y)\, h_{m,n}(X,Y)$$

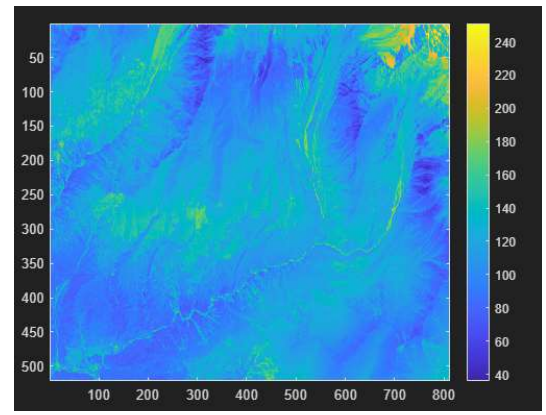
density dist

concentration dist

altitude dist







Literature:

[1]. *Mount Ontake Landslide Simulation by the Cellular Automata Model, S. Di Gregorio, R. Rongo, C. Siciliano, M.Sorriso-Valvo, W. Spataro,1998*

[2]. *E. Žic et al.: A model of mudflow propagation downstream from the Grohovo landslide near Rijeka*

[3]. *Assessment report of Tragedy on 3rd of August 2023 by National Envirvorment agency of Georgia*

[4]. *Mathematical modeling of mudflow dynamics, Obgadze, Kipiani, Gurgenidze 2021*

Dziękuje za uwagę?
雲 ですが、 なにか？
Thank you for the attention?
გმადლობთ ყურადღებისთვის?