Conformal Parametrization of the Pion Vector Form Factor

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Challenges to the Standard Model

High energy regime

- quarks as weakly interacting particles
- series expansion of α_S converges
- perturbation theory manageable
  → QCD can be applied

Low energy regime

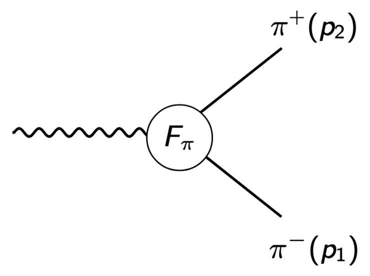
- large coupling between quarks
  → form color neutral bound states (hadrons)
- series expansion of α_S does not converge
- perturbation theory not possible
  → need alternative ansatz to handle these confined states

- SM describes majority of phenomena of modern particle physics
- BUT challenged by anomalies and experimental observations (i.e. contradictions between experimental results and theory predictions)

Pion Vector Form Factor

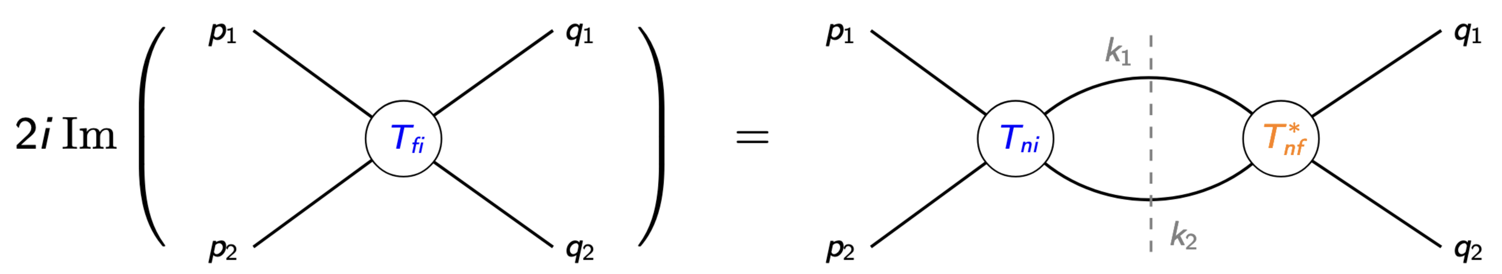
- matrix element for pion vector form factor

$$\mathcal{M} = \pm e\varepsilon^{\mu}\langle\pi(p_1)|J_\mu|\pi(p_2)\rangle = \pm e\varepsilon^{\mu}F_\pi(q^2)(p_1+p_2)_\mu$$

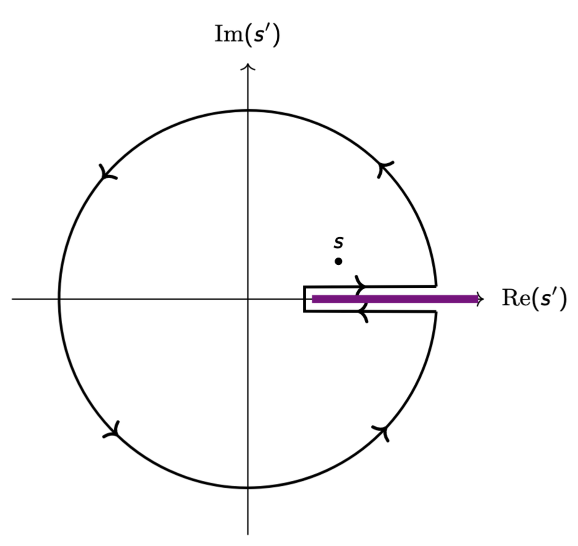


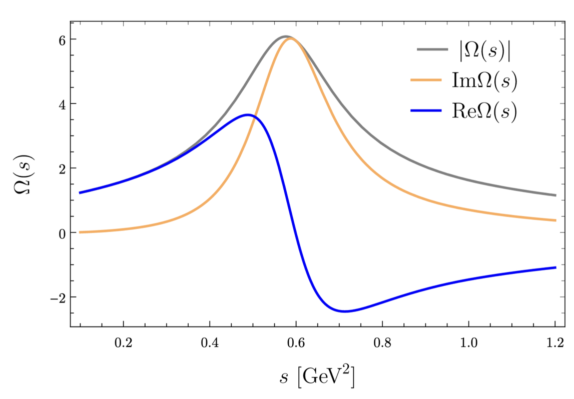
- isovector part of two quark current $J_\mu = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)$
- unitarity (probability conservation)

$$\mathrm{disc}T_{fi} = T_{fi} - T^*_{if} = i\sum_n \int \mathrm{d}\rho_n (2\pi)^4\delta^{(4)}(p_i - k_n) T_{ni}T^*_{nf}$$



- analyticity





- dispersion relation

$$F(s+i\varepsilon) = \frac{1}{2\pi i}\int_{s_\mathrm{thr}}^{\infty}\frac{\mathrm{disc}F(x)}{x-s-i\varepsilon}\mathrm{d}x$$

- discontinuity related to full form factor

$$\mathrm{disc}F_\pi(t) = 2iF_\pi(t)e^{-i\delta(t)}\sin(\delta(t))\Theta(t-4M_\pi^2)$$

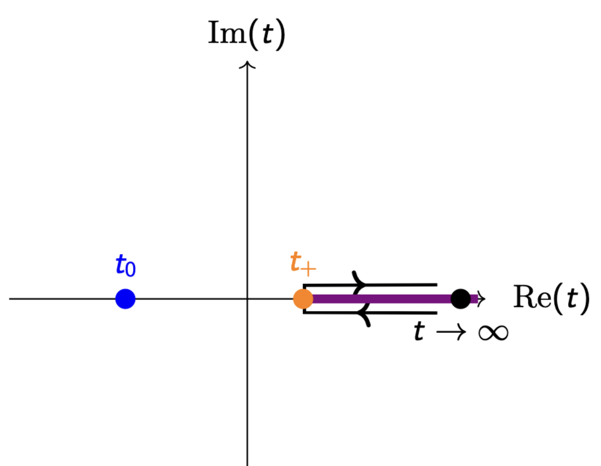
- solved by real-valued polynomial $P(t)$ and Omnès function $\Omega(t)$ (resummation of rescattering involving pions)

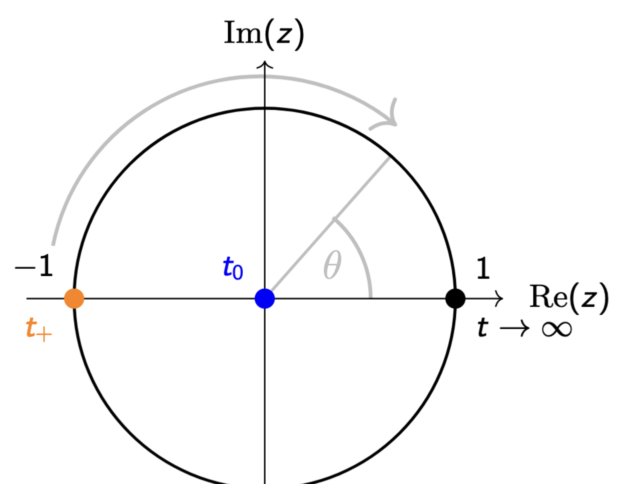
$$F_\pi(t) = P(t)\Omega(t) \qquad \Omega(t) = \exp\left(\frac{t}{\pi}\int_{t_{thr}}^{\infty}\frac{\delta(s)}{s(s-t)}\mathrm{d}s\right)$$

Conformal Transformation

- conformal invariance: angles between intersecting oriented curves preserved in conformal transformations [1]

$$t(z,t_0) = \frac{-4t_+z + t_0(z+1)^2}{(z-1)^2} \qquad z(t,t_0) = \frac{\sqrt{t_+-t}-\sqrt{t_+-t_0}}{\sqrt{t_+-t}+\sqrt{t_+-t_0}}$$





Dispersive Bound

- two-point Green's function in momentum space

$$\Pi_J^{\mu\nu}(q^2) \equiv \mathrm{i}\int \mathrm{d}^4x e^{iqx}\langle 0|TJ^\mu(x)J^{\dagger\nu}(0)|0\rangle$$

- n-times subtracted dispersion relation [2]

$$\chi(Q^2) = \frac{1}{n!}\frac{\partial^n\Pi^\mathrm{T}(Q^2)}{\partial(Q^2)^n} = \frac{1}{\pi}\int_0^\infty \mathrm{d}t\frac{\mathrm{Im}\Pi^\mathrm{T}(t)}{(t-Q^2)^{n+1}} \quad\Rightarrow\quad \frac{1}{\pi\chi(Q^2)}\int_0^\infty \mathrm{d}t\frac{\mathrm{Im}\Pi^\mathrm{T}(t)}{(t-Q^2)^{n+1}} = 1$$

- compute $\mathrm{Im}\Pi^\mathrm{T}(t)$ by both perturbative QCD and by using hadronic states
- *quark-hadron duality:* $\chi_\mathrm{OPE}(Q^2) = \chi_\mathrm{had}(Q^2)$
- describing dispersive bound by a product of form factor $F_\pi(t)$ and outer function $\phi(t)$ for $X = \pi^-\pi^+$

$$\frac{1}{\pi}\int_{t_+}^{\infty}\mathrm{d}t\left|\frac{\mathrm{d}z(t)}{\mathrm{d}t}\right|\cdot|\phi(t)F_\pi(t)|^2 \le 1\,, \qquad \frac{1}{2\pi i}\int_C\frac{\mathrm{d}z}{z}|\phi(z)F_\pi(z)|^2 \le 1$$

- expansion of $\phi(z)F_\pi(z)$ about $z=0$ leads to ansatz

$$F_\pi(z) = \frac{1}{\phi(z)}\sum_{n=0}^\infty a_n z^n\,, \qquad \sum_n |a_n|^2 \le 1$$

Manipulating Series Expansion

- $F_\pi(z)$: finite for $z=-1$, zero for $z=+1$
- factorize weight function $W(z)$ and orthonormal polynomials $P_n(z)$ of degree n

$$\sum_{n=0}^\infty a_n z^n \to W(z)\sum_{n=0}^\infty b_n P_n(z)$$

– transferring bounds for a_n to b_n

$$F(z) = \frac{W(z)}{\phi(z)}\sum_{n=0}^\infty b_n P_n(z) \qquad \sum_{n=0}^\infty |b_n|^2 \le 1$$

- construct $W(z)$ to correct singularity of $\phi(z)$ and include asymptotic behavior
  1. $z=+1$: factorize $(1-z)^{5/2}$, leaving $(1-z)^2 \propto 1/t$
  2. $z=-1$: factorize $(1+z)^2$ for finite F_π
- putting together $W(z) = (1+z)^2(1-z)^{5/2}$

Implementing Constraints

- normalization

$$F_\pi(0) = \frac{W(0)}{\phi(0)}\left(b_0P_0(0) + \sum_{n=1} b_nP_n(0)\right) = 1$$

- threshold behavior: relative p-wave ⇒ imaginary part $\propto (t_+-t)^{(2l+1)/2}$
- asymptotic behavior $z\to -1$: expand $F_\pi(t)$ in terms of $\sqrt{t_+-t}$

$$\mathrm{Im}F_\pi(t) = c_1\cdot 0 + c_2\cdot\mathrm{Im}(\sqrt{t_+-t}) + c_3\cdot 0 + \mathcal{O}(\mathrm{Im}((t_+-t)^{3/2}))$$

⇒ remove leading imaginary term

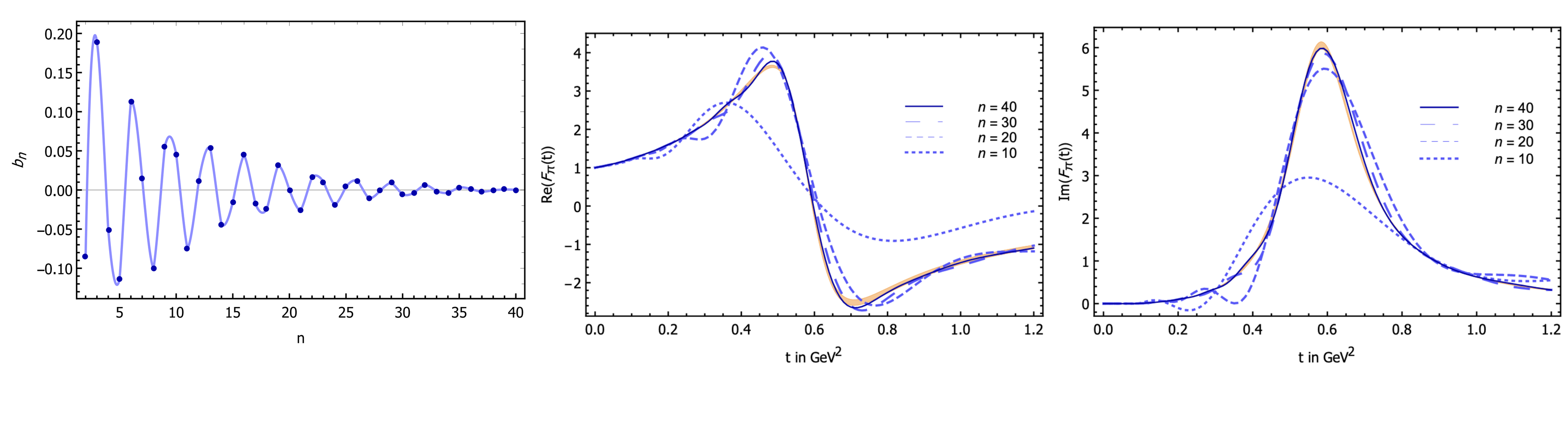
- implementation of threshold behavior by forcing $\quad (\mathrm{d}F_\pi(y)/\mathrm{d}y)|_{y_\mathrm{thr}} = 0$

Results

- data constructed using Omnès function
- minimize $\chi^2 = \chi^2_\mathrm{Re} + \chi^2_\mathrm{Im}$ with estimated error $\Delta\Omega_\mathrm{Re/Im}$

$$\chi^2_\mathrm{Re} = \sum_i\left(\frac{\mathrm{Re}(F_\pi(x_i)) - \mathrm{Re}(\Omega(x_i))}{\Delta\Omega_\mathrm{Re}(x_i)}\right)^2, \qquad \Delta\Omega_\mathrm{Re} = \left|\frac{\mathrm{Re}(\Omega(\delta+\Delta\delta)) - \mathrm{Re}(\Omega(\delta-\Delta\delta))}{2}\right|$$

- best fit: $\chi^2_\mathrm{red} = 1.29$ with $n=40$



References

[1] W. W. Buck et al. "New constraints on dispersive form factor parametrizations from the timelike region". In: *Phys. Rev. D* (5 July 1998).

[2] B. G. C. G. Boyd et al. "Precision corrections to dispersive bounds on form-factors". In: *Phys. Rev. D* (1997 [hep-ph/9705252]). arXiv: hep-ph/9705252.

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