PBH overproduction bounds on LISA PT target sources

Master's Dissertation Presentation @ ICPS 2024 — Georgia

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Real Sociedad Española de Física

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  - First space-based GW observatory
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  - SSB of nature symmetries
  - First order and second order PTs
  - SBGW[1] production [LISA detection]
- **Primordial Black Holes (PBH) formation**
  - Supercooled cosmological PTs (α, β)
  - Good dark matter (DM) candidate $f_{\mathrm{PBH}} \neq 0$

OUR AIM: **constrain** LISA PT target sources **parameter space** (α, β) through **PBH overproduction bounds** $f_{\mathrm{PBH}} \geq 1$ formed by CPTs.

[1] Stochastic Background of Gravitational Waves

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Introduction

Phase transitions: physical process of transition from one state of a medium and another

- e.g. solid-liquid transitions: ice melting into water @ Critical temperature T_C
- e.g. ferromagnetic-paramagnetic transition in magnetic materials @ Curie temperature T_C

Cosmological phase transition: physical process where the overall state of matter changes together across the whole universe

- e.g. EW[2] phase transition when the Higgs mechanism was activated @ Electroweak scale T_{EW}
- e.g. BSM[3] cosmological phase transitions

---

[2] Electroweak

[3] Beyond the Standard Model

Introduction

Cosmological phase transition are **driven** by a **scalar field** ϕ with potential $V(\phi)$

- Spontaneous Symmetry Breaking (SSB):
  Underlying symmetries of nature not present in the vacuum (unified field gauge theories symmetry group is larger than that of the vacuum)
  - e.g. The SM[4]: $SU(3)_C \times SU_W(2) \times U_Y(1) \xrightarrow{\text{SSB}} SU(3)_\mathrm{C} \times U_Q(1)$
- High temperature symmetry restoration
  - Effective potential temperature dependence $V_{\text{eff}}(\phi, T)$
  - Vacuum state[5] $\langle\phi(T)\rangle$[6] depends on T

[4] Standard Model.

[5] State of lowest energy i.e. minimum of the potential $V(\phi, T)$

[6] QFT are built on excitations over vacuum states.

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Cosmological scenario

Simple example of a potential describing a PT

$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 + \frac{\lambda(T)}{4}\phi^4$$

Hot big bang theory:

- universe initially @ very high T: universe in the symmetric phase $\langle\phi(T)\rangle = 0$
- $T = T_c$, $\phi(T) = 0$ metastable
- $T < T_c$, $\phi(T) = \pm\sigma$ stable
- PT proceeds and universe transitions to the broken phase $\langle\phi(T)\rangle = \pm\sigma$

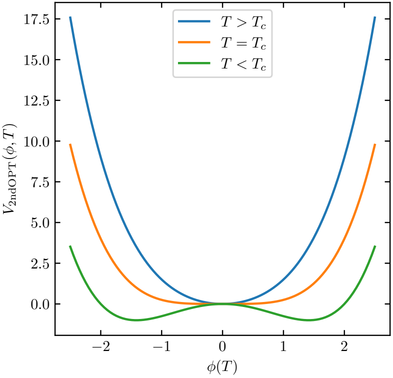

Figure 1: Example of a potential that implements SSB and sources a cosmological phase transition.

Types of cosmic PTs

- **First-order phase transitions** (1stOPTs):
  - baryogenesis uses and SBGW production
- **Second-order phase transitions** (2ndOPTs)
  - new inflationary models e.g. slow roll-down models
- **Cross-over transitions** (2ndOPT with finite correlation length ξ)
  - e.g. EW phase transition in the SM

Second order PTs

Simple example of a potential describing a 2ndOPT

$$V(\phi, T)^a = D(T^2 - T_0^2)\phi^2 + \frac{\lambda(T)}{4}\phi^4$$

[a] SM Higgs' potential-like

- Critical temperature $T_c = T_0$
- **NO** barrier between symmetric and broken phases
- PT achieved by thermal fluctuation for a field in the origin
- PT starts at T_c and occurs smoothly: no latent heat

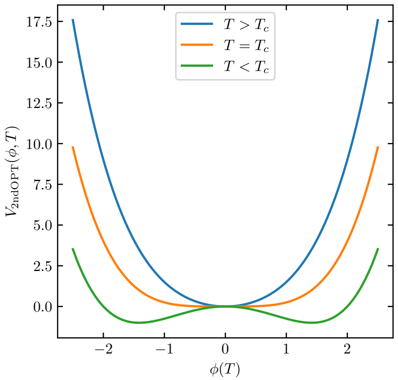

Figure 2: Example of a potential that implements SSB and sources a second order phase transition.

False and true vacuum

The scalar field ϕ driving the transition has a potential $V(\phi, T)$ with

- **False vacuum[7]:** locally stable vacuum but not most stable possible ground state: metastable vacuum state
- **True vacuum[8]:** globally stable vacuum

[7] Vacuum in this sense means minimum of the potential.

[8] Physical vacuum state we live in, and from which we build QFTs.

First order PTs

Simple Example of a potential describing a 1stOPT

$$V(\phi, T)^a = D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\lambda(T)}{4}\phi^4$$

[a] Cubic term provided by contribution from bosonic fields.

- Critical temperature $T_c > T_0$
- **Barrier** appears: **supercooling**
- PT achieved by **tunnelling**: quantum mechanical or thermal[b]
- PT effectively starts at T_n through **bubble nucleation**[c]

[b] Thermal tunnelling is just tunnelling at finite temperature whereas quantum tunnelling is tunnelling at zero temperature.

[c] A useful analogue is boiling water in which bubbles of steam form and expand as they rise to the surface.

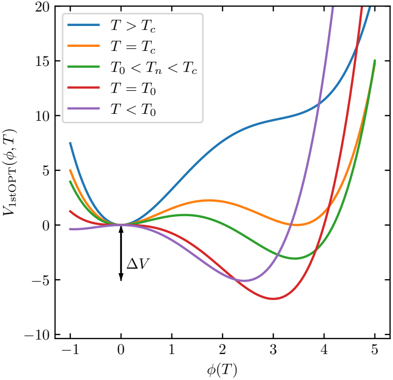

Figure 3: Example of a potential that implements SSB and sources a first order phase transition.

False and true vacuum

The scalar field ϕ driving the transition has a potential $V(\phi, T)$ with

- **False vacuum:** locally stable vacuum
- **True vacuum:** globally stable vacuum

False vacuum decay $\equiv$ 1st Order Cosmological Phase Transition

$$\text{False vacuum } \langle\phi(T)\rangle = 0 \xrightarrow[\text{Bubble nucleation}]{\text{Tunnelling}} \text{True vacuum } \langle\phi(T)\rangle = \sigma$$

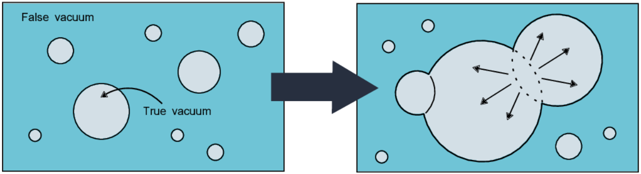

Figure 4: Schematic illustration of a first-order phase transition. Bubbles of the true vacuum are nucleated in the false vacuum. These expand, and collide. GW are sourced both by the bubble collisions themselves, and by the overlapping sound shells after the bubbles have merged.

Supercooled 1stOPTs

Universe energy budget

$$\rho_{\text{tot}} = \rho_V + \rho_R \quad \text{where} \quad \rho_R = \rho_{\text{wall}} + \rho_{\text{plasma}} + \rho_{\text{scalar}} + \rho_{\text{th}} + \rho_{\text{cool}}$$

- vacuum component ρ_V initially given by latent heat ΔV
- radiation component ρ_R initially pre-existing super-cooled plasma

Total false vacuum decay rate per unit volume

$$\Gamma_V(t) = \max(\Gamma_{\text{QT}}(t), \Gamma_{\text{therm}}(t))$$

- QM tunneling contribution dominates for vacuum transitions and @ very low T for thermal transitions with barrier @ $T = 0$
- Thermal tunneling dominates @ high T

Supercooled 1stOPTs

False vacuum decay rate $\equiv$ Tunneling probability $\equiv$ Bubble nucleation rate

Nucleation rate frequently taken[9] to be $\Gamma(t) = A(t)e^{-B(t)}$

Exponential nucleation rate per unit of volume

$$\Gamma_V(t)^a = \Gamma_0 e^{\beta t} \quad \text{with} \quad \Gamma_0 = H^4(T_n)e^{-\beta t_n}$$

[a] This should be viewed as a Taylor-expansion of the bounce action around t_n at first order.

- Bubble nucleation instantaneous criterion

$$T = T_n \longleftrightarrow \Gamma_V(t) = \Gamma_0 = H^4(T_n)$$

[9] False vacuum decay theory from finite temperature field theory.

Supercooled 1stOPTs paramater space: strength α and duration β^{-1}

- The "**strength**" (energy released) of the 1stOPTs is quantified by the latent heat

Strength of the 1stOPT — Latent Heat Parameter α

$$\alpha \equiv \left.\frac{\rho_V}{\rho_R}\right|_{T=T_n} = \left(\frac{T_{\rm eq}}{T_{\rm n}}\right)^4 \equiv e^{4N_e}$$

- The "**duration**" (average time for bubbles to percolate[10]) of the 1stOPTs is defined through the nucleation rate

Duration of the 1stOPT β^{-1}

$$\beta \equiv \frac{1}{\Gamma_V}\frac{\mathrm{d}\Gamma_V}{\mathrm{d}t}$$

[10] Percolation time is the time when most of the vacuum energy has been converted into radiation

PBH formation

Process of PBH formation from supercooled 1stOPT:

- QM or thermal tunnelling starts bubble nucleation
- Vacuum energy ρ_V converted into a mixture of relativistic species (bubble walls, relativistic kinetic and thermal energy, relativistic scalar waves...)

Time evolution of the energy densities (continuity equation)

$$\rho_R(t;t_{n_i})^{a} + 4H\rho_R(t;t_{n_i}) = -\dot{\rho}_V(t;t_{n_i}) \quad \text{with} \quad H = \sqrt{\frac{\rho_V + \rho_R}{3M_{\mathrm{Pl}}^2}}$$

[a] t_{n_i} is a free parameter setting the time at which the first bubble is nucleated in a given causal patch.

- Any delay of percolation in a specific causal patch[11] necessarily generates an overdensity of ρ_R wrt to the background.

[11] Synonym for Hubble patch: region of the universe surrounding an observer for which $v_{\mathrm{rec}} > c$

PBH formation

Process of PBH formation from supercooled 1stOPT:

- Any delay of percolation in a specific causal patch[12] necessarily generates an overdensity of ρ_R wrt to the background.

Radiation over-density of a lately-nucleated Hubble patch

$$\delta(t; t_{n_i}) \equiv \frac{\rho_R^{\mathrm{late}}(t; t_{n_i}) - \rho_R^{\mathrm{bkg}}(t)}{\rho_R^{\mathrm{bkg}}(t)}$$

- if over-density of a lately nucleated patch reaches a threshold[13]

PBH formation criterion (based on Hoop's Conjecture)

If $\delta(t; t_{n_i}) > \delta_c = 0.45$ then this late Hubble patch collapses into a PBH[a]

[a] Regardless of the Particle Physics model used, model independent result!

[12] Synonym for Hubble patch: region of the universe surrounding an observer for which $v_{\mathrm{rec}} > c$

[13] Found by simulations based on Hoop's conjecture [1, 2]

Simulation of PBH formation from a supercooled 1stOPT

- Solving system of **coupled ordinary 1st order IDEs**[14] for $\rho_R(t)$, $a(t)$

Time evolution of the energy densities (continuity equation)

$$\rho_R(t;t_{n_i}) + 4H\rho_R(t;t_{n_i}) = -\dot{\rho}_V(t;t_{n_i}) \quad \text{with} \quad H = \sqrt{\frac{\rho_V + \rho_R}{3M_{\rm Pl}^2}}$$

Vacuum energy density evolution

$$\rho_V(t;t_{n_i}) = F(t;t_{n_i})\Delta V \quad ; \quad F(t;t_{n_i}) = \exp\left\{-\int_{t_{n_i}}^{t} \mathrm{d}t' \Gamma_V(t') a(t')^3 \frac{4\pi}{3} r^3(t;t')\right\}$$

- Can be carefully transformed into a set of **7 coupled 1st order ODEs**[15] — easy to solve numerically!

[14] Integro-Differential Equations

[15] Ordinary differential equations.

Simulation of PBH formation from a supercooled 1stOPT

Full expression of the system of coupled ordinary 1st order IDEs[16]

$$\frac{\mathrm{d}\rho_R(t;t_{n_i})}{\mathrm{d}t} + 4\rho_R(t;t_{n_i})\times$$

$$\times\sqrt{\frac{1}{3M_{\mathrm{Pl}}^2}\left(\rho_R(t;t_{n_i}) + \Delta V \exp\left\{-\left(\frac{(1+\alpha^{-1})\Delta V}{3M_{\mathrm{Pl}}^2}\right)^2 \int_{t_{n_i}}^{t} \mathrm{d}t' e^{\beta(t'-t_n)} a(t')^3 \frac{4\pi}{3}\left(\int_{t'}^{t} \mathrm{d}\tilde{t}\,\frac{v_w(\tilde{t})}{a(\tilde{t})}\right)^3\right\}\right)}$$

$$= \Delta V \frac{v_w(t)}{a(t)} \exp\left\{-\left(\frac{(1+\alpha^{-1})\Delta V}{3M_{\mathrm{Pl}}^2}\right)^2 \int_{t_{n_i}}^{t} \mathrm{d}t' e^{\beta(t'-t_n)} a(t')^3 \frac{4\pi}{3}\left(\int_{t'}^{t} \mathrm{d}\tilde{t}\,\frac{v_w(\tilde{t})}{a(\tilde{t})}\right)^3\right\}$$

$$\times\left(\frac{(1+\alpha^{-1})\Delta V}{3M_{\mathrm{Pl}}^2}\right)^2 \int_{t_{n_i}}^{t} \mathrm{d}t' e^{\beta(t'-t_n)} a(t')^3 4\pi\left(\int_{t'}^{t} \mathrm{d}\tilde{t}\,\frac{v_w(\tilde{t})}{a(\tilde{t})}\right)^2$$

$$\frac{1}{a(t)}\frac{\mathrm{d}a(t)}{\mathrm{d}t} =$$

$$\sqrt{\frac{1}{3M_{\mathrm{Pl}}^2}\left(\rho_R(t;t_{n_i}) + \Delta V \exp\left\{-\left(\frac{(1+\alpha^{-1})\Delta V}{3M_{\mathrm{Pl}}^2}\right)^2 \int_{t_{n_i}}^{t} \mathrm{d}t' e^{\beta(t'-t_n)} a(t')^3 \frac{4\pi}{3}\left(\int_{t'}^{t} \mathrm{d}\tilde{t}\,\frac{v_w(\tilde{t})}{a(\tilde{t})}\right)^3\right\}\right)}$$

[16] Integro-Differential equations.

Simulation of PBH formation from a supercooled 1stOPT

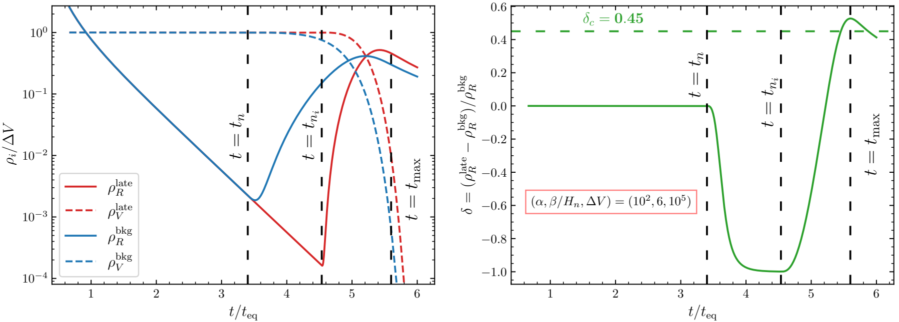

Figure 5: Time evolution of vacuum (dotted) and radiation (solid) energy density during a supercooled 1stOPT.

Simulation of PBH formation from a supercooled 1stOPT

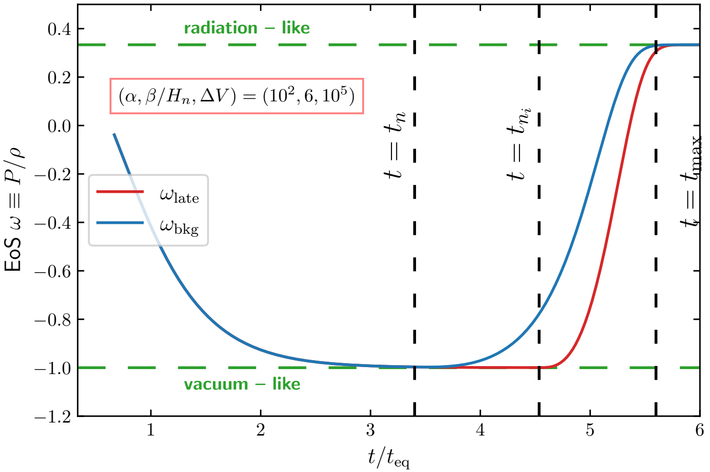

Figure 6: Effective equation of state (EoS) ω during a supercooled 1stOPT where the universe changes from a vacuum-like EoS $\omega = -1$ to a radiation EoS with $\omega = 1/3$.

PBHs as DM candidate

- To find the fraction of DM in the form of PBHs we first need

Probability of survival

$$P_{\rm surv}(t_{n_i}; t_{\rm max}) = \exp\left\{ -\int_{t_c}^{t_{n_i}} \mathrm{d}t' \Gamma_V(t') a(t')^3 V(t'; t_{\rm max}) \right\}$$

- then the probability of a late Hubble patch to collapse into a PBH

Probability of collapse

$$P_{\rm coll} \equiv P_{\rm surv}(t_{n_i}^{\rm PBH}; t_{\rm max})$$

Fraction of DM in the form of PBHs

$$f_{\rm PBH} \approx \left(\frac{P_{\rm coll}}{2.45 \times 10^{-12}} \right) \left(\frac{T_{\rm eq}}{500\ \mathrm{GeV}} \right)$$

GW signatures: types of sources

GW sources during 1stOPT can be split into:

- Collision of bubble walls
- Plasma sound waves
- Plasma turbulent flows

These sources may coexist and form the dominant source at different stages of the 1stOPT

Total SBGW is approx. a linear superposition of GW sources:

Total SBGW during a 1stOPT

$$\Omega_{\rm gw} = \Omega_\phi + \Omega_{\rm sw} + \Omega_{\rm turb}$$

GW signatures: Scalar field ϕ

The collision of bubble walls generates GW

- when bubbles collide spherical symmetry is broken
- shear-stress in the gradients of ϕ source GW
- short-lived source of GW
- Two-possible models:
    - Envelope approximation
    - Bulk-flow model

GW power spectrum contribution from the scalar field ϕ:

$$\Omega_{\mathrm{env},0}(f)^{a} = \Delta_{\mathrm{env}}(v_w)\kappa^2 \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\alpha}{\alpha+1}\right) \left(\frac{100}{g_*}\right)^{1/3} P_{\mathrm{env}}(f)$$

[a] 0 and * subindices correspond to quantities evaluated today and at the time of GW production.

GW signatures: acoustic waves

In thermal PTs:

- fluid shells develop around the bubble wall
- these propagate in the form of **sound waves**
- shear-stress in the plasma sound waves generates GW
- long-lasting source of GW
- decay through shocks

Plasma sound waves generate GW:

GW power spectrum contribution from the scalar waves

$$\frac{\mathrm{d}\Omega_{\mathrm{sw},0}}{\mathrm{d}\log(f)} = 7.34 \times 10^{-5} \Gamma^2 \overline{U}_f^4 \left(\frac{100}{g_*}\right)^{1/3} (H_* \tau_v) \left(\frac{H_* R_*}{c_s}\right) \tilde{\Omega}_{\mathrm{gw}} C \left(\frac{f}{f_{\mathrm{sw},0}}\right)$$

GW signatures: turbulence

In thermal PTs:

- fluid turbulent flows
- magnetic field: magnetohydrodynamic (MHD) turbulence
- also vorticity modes from decay of scalar waves

Kinetic and MHD turbulence are sources of GW:

GW power spectrum contribution from turbulence

$$\frac{\mathrm{d}\Omega_{\mathrm{turb},0}}{\mathrm{d}\log(f)} = 7.29 \times 10^{-4} \left(\frac{H_*}{\beta}\right) \left(\frac{\kappa_{\mathrm{turb}}\alpha_\theta}{1+\alpha_\theta}\right)^{3/2} \left(\frac{100}{g_*}\right)^{1/3} v_w S_{\mathrm{turb}}(f)$$

LISA PT sources parameter space

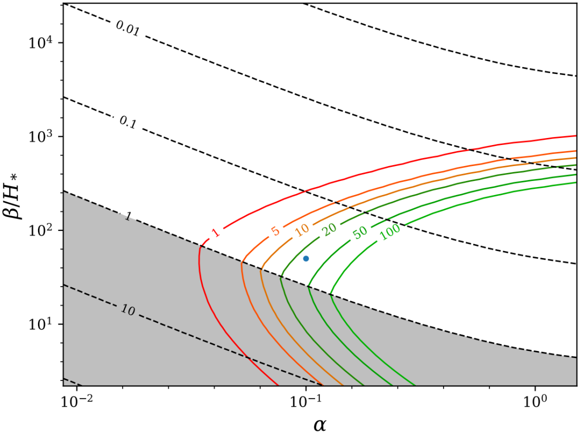

Figure 7: LISA PT target sources parameter space (α, β). The coloured lines show the SNR. The dotted straight lines are the contours of the fluid turnover time quantifying the effect of turbulence. In the gray shaded region the decay of sound waves into turbulence is less important than the Hubble damping and the SNR curve reflects this effect

Conclusions

Summary

- Cosmological PTs are theoretical large scale vast phemomena
- 1stOPT can produce PBH[17] in lately nucleated Hubble patch
- PBH as DM candidate can not exceed $f_{\mathrm{PBH}} \geq 1$
- this overproduction bound constrains LISA PT target sources parameter space (α, β)

Outlook and future work

Constantly advancing fields:

- Better PBH collapse dynamics understanding & GW modelling

Inspired by Dvali's Monday talk:

- PBH evaporation implementation?

[17] Model independently!

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